SuperGrads Study Material

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QUANTITATIVE ABILITY

NUMBER SYSTEM

- Number Systems is the most important topic in the quantitative section.
- It is a very vast topic and a significant number of questions appear in CAT every year from this section.
- Learning simple tricks like divisibility rules, HCF and LCM, prime number and remainder theorems can help improve the score drastically.
- This document presents best short cuts which makes this topic easy and helps you perform better.

Concept 01
HCF and LCM
- HCF * LCM of two numbers = Product of two numbers
- The greatest number dividing a, b and c leaving remainders of x_1, x_2 and x_3 is the HCF of (a - x_1), (b - x_2) and (c - x_3).
- The greatest number dividing a, b and c (a < b < c) leaving the same remainder each time is the HCF of (c-b), (c - a), (b - a).
- If a number, N, is divisible by X and Y and HCF(X, Y) = 1. Then, N is divisible by X*Y

Concept 02
Prime and Composite Numbers
- Prime numbers are numbers with only two factors, 1 and the number itself.
- Composite numbers are numbers with more than 2 factors.
  Examples are 4, 6, 8, 9.
- 0 and 1 are neither composite nor prime.
- There are 25 prime numbers less than 100

Concept 03
Properties of Prime numbers
- To check if n is a prime number, list all prime factors less than or equal to √n. If none of the prime factors can divide n then n is a prime number.
- For any integer a and prime number p, $a^p - a$ is always divisible by p
- All prime numbers greater than 2 and 3 can be written in the form of 6k + 1 or 6k - 1
- If a and b are co-prime then $a^{(b-1)}$ mod b = 1.

Concept 04
Theorems on Prime numbers
Fermat's Theorem:
Remainder of $a^{(p-1)}$ when divided by p is 1, where p is a prime

Wilson's Theorem:
Remainder when (p-1)! Is divided by p is (p-1) where p is a prime

Theorems on Prime numbers

Remainder Theorem
- If a, b, c are the prime factors of N such that N= $a^p * b^q * c^r$

Then the number of numbers less than N and co-prime to N is
$\phi(N) = N\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\left(1 - \frac{1}{c}\right)$.
This function is known as the Euler's totient function.

Euler's theorem
- If M and N are co-prime to each other then remainder when $M^{\phi(N)}$ is divided by N is 1.

Concept 05
- Highest power of n in m! is $\left[\frac{m}{n}\right] + \left[\frac{m}{n^2}\right] + \left[\frac{m}{n^2}\right]$ +…..

  Ex: Highest power of 7 in 100! = $\left[\frac{100}{7}\right] + \left[\frac{100}{49}\right]$ = 16
- To find the number of zeroes in n! find the highest power of 5 in n!
- If all possible permutations of n distinct digits are added together the sum = (n - 1)! * (sum of n digits) * (11111... n times)

Concept 06
- If the number can be represented as $N = a^p * b^q * c^r$. . . then number of factors the is (p+1) * (q+1) * (r+1)
- Sum of the factors = $\frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1}$
- If the number of factors are odd then N is a perfect square.
- If there are n factors, then the number of pairs of factors would be $\frac{n}{2}$. If N is a perfect square then number of pairs (including the square root) is $\frac{(n+1)}{2}$

If the number can be expressed as $N = 2^p * a^q * b^r$ . . . where the power of 2 is p and a, b are prime numbers
- Then the number of even factors of N = p (1 + q) (1 + r) . . .
- The number of odd factors of N = (1 + q) (1 + r)…

Concept 07
Number of positive integral solutions of the equation $x^2 - y^2 = k$ is given by

- $\frac{\text{Total number of factors of k}}{2}$ (If k is odd but not a perfect square)
- $\frac{(\text{Total number of factors of k}) - 1}{2}$ (If k is odd and a perfect square)
- $\frac{\text{Total number of factors of } \frac{k}{4}}{2}$ (If k is even and not a perfect square)
- $\frac{\left(\text{Total number of factors of } \frac{k}{4}\right) - 1}{2}$ (If it is even and a perfect square)

Concept 08
- Number of digits in ab = [b $\log_m(a)$] + 1 ; where m is the base of the number and [.] denotes greatest integer function
- Even number which is not a multiple of 4, can never be expressed as a difference of 2 perfect squares.
- Sum of first n odd numbers is n^2
- Sum of first n even numbers is n(n + 1)
- The product of the factors of N is given by $N^{\frac{a}{2}}$, where a is the number of factors

Concept 09
- The last two digits of a^2, $(50 - a)^2$, $(50 + a)^2$, $(100 - a)^2$ are same.
- If the number is written as 210n

When n is odd, the last 2 digits are 24.
When n is even, the last 2 digits are 76.

Concept 10
Divisibility
- Divisibility by 2: Last digit divisible by 2
- Divisibility by 4: Last two digits divisible by 4
- Divisibility by 8: Last three digits divisible by 8
- Divisibility by 16: Last four digit divisible by 16

Divisibility
- Divisibility by 3: Sum of digits divisible by 3
- Divisibility by 9: Sum of digits divisible by 9
- Divisibility by 27: Sum of blocks of 3 (taken right to left) divisible by 27
- Divisibility by 7: Remove the last digit, double it and subtract it from the truncated original number. Check if number is divisible by 7
- Divisibility by 11: (sum of odd digits) - (sum of even digits) should be 0 or divisible by 11

Concept 11
Divisibility properties
- For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.
- The equation $a^n - b^n$ is always divisible by a - b. If n is even it is divisible by a + b. If n is odd it is not divisible by a + b.
- The equation $a^n + b^n$, is divisible by a + b if n is odd. If n is even it is not divisible by a + b.

- Converting from decimal to base b. Let R_1, R_2 . . . be the remainders left after repeatedly dividing the number with b. Hence, the number in base b is given by ... R_2R_1.
- Converting from base b to decimal - multiply each digit of the number with a power of b starting with the rightmost digit and b^0.
- A decimal number is divisible by b-1 only if the sum of the digits of the number when written in base b are divisible by b - 1.

Concept 12
Cyclicity
- To find the last digit of an find the cyclicity of a. For Ex. if a = 2, we see that
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$

Hence, the last digit of 2 repeats after every 4^{th} power. Hence cyclicity of 2 = 4. Hence if we have to find the last digit of a^n,

The steps are:
1. Find the cyclicity of a, say it is x
2. Find the remainder when n is divided by x, say remainder r
3. Find a^r if r > 0 and a^x when r = 0

Concept 13
- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$.