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## IPMAT INDORE 2019



By Toprankers

## QA Short Answer (SA)

1. The sum of the interior angles of a convex $n$-sided polygon is less than $2019^{\circ}$. The maximum possible value of n is

## Solutions:

Sum of interior angle of on $n-$ sided polygon $=(n-2) \times 180^{\circ}$
As per question,
$(\mathrm{n}-2) \times 180<2019$
$(\mathrm{n}-2)<\frac{2019}{180}$
$(\mathrm{n}-2)<11$
n $<13.21$
$\therefore$ Maximum value of n will be 13 .
2. Suppose that $a, b$, and $c$ are real numbers greater than 1 . Then the value of $\frac{1}{1+\log _{a^{2} b} \frac{c}{a}}+\frac{1}{1+\log _{b^{2} c} \frac{a}{b}}$ $+\frac{1}{1+\log _{\mathrm{c}^{2} \mathrm{a}} \frac{\mathrm{b}}{\mathrm{c}}}$ is

## Solutions:

$\frac{1}{1+\log _{a^{2} b} \frac{c}{a}}+\frac{1}{1+\log _{b^{2} c} \frac{a}{b}}+\frac{1}{1+\log _{c^{2} a} \frac{b}{c}}$
1 st term in the expression $=\frac{1}{1+\log _{a^{2} b} \frac{c}{a}}$

$$
\begin{aligned}
& =\frac{1}{1+\frac{\log \frac{c}{a}}{\log a^{2} b}}=\frac{\log a^{2} b}{\log a^{2} b+\log \frac{c}{a}} \quad\left(\because \log _{y} x=\frac{\log x}{\log y}\right) \\
& =\frac{\log a^{2} b}{\log \left(a^{2} b \cdot \frac{c}{a}\right)}=\frac{\log a^{2} b}{\log a b c} \quad(\because \log x y z=\log x+\log y+\log z)
\end{aligned}
$$

$\therefore$ The whole expression will become
$\frac{\log a^{2} b}{\log a b c}+\frac{\log b^{2} c}{\log a b c}+\frac{\log c^{2} a}{\log a b c}$
$\frac{\log (a b c)^{3}}{\log a b c}=3 \quad\left(\because \log x^{a}=\log x\right)$
3. A real-valued function $f$ satisfies the relation $f(x) f(y)=f(2 x y+3)+3 f(x+y)-3 f(y)+6 y$, for all real numbers $x$ and $y$, then the value of $f(8)$ is

## Solutions:

Putting $x=y=0$, we get
$f(0) . f(0)=f(3)+3 f(0)-3 f(0)+0$
$[\mathrm{f}(0)]^{2}=\mathrm{f}(3)$
Putting $x=0, y=3$, we get
$f(0) \cdot f(3)=f(3)+3 f(3)-3 f(3)+18$
$[\mathrm{f}(0)]^{3}=\mathrm{f}(0)^{2}+18$
Let $\mathrm{f}(0)=\mathrm{x}$
So, $x^{3}=x^{2}+18$
Solving this equation we get $x=f(0)=3$
Substituting $\mathrm{f}(0)=3$ in equation (i) we get
$\mathrm{f}(3)=9$
Now putting $\mathrm{x}=0, \mathrm{y}=8$ we get
$f(0) . f(8)=f(3)+3 f(8)-3 f(8)+48$
Substituting the value of $f(0)$ and $f(3)$ we get $f(8)=19$
4. Let $\mathrm{A}, \mathrm{B}, \mathrm{C}$ be three $4 \times 4$ matrices such that $\operatorname{det} \mathrm{A}=5, \operatorname{det} \mathrm{~B}=-3$, and $\operatorname{det} \mathrm{C}=\frac{1}{2}$. Then the $\operatorname{det}\left(2 \mathrm{AB}^{-1} \mathrm{C}^{3} \mathrm{~B}^{\mathrm{T}}\right)$ is

## Solutions:

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\(\operatorname{det}\left(2 \mathrm{AB}^{-1} \mathrm{C}^{3} \mathrm{~B}^{\mathrm{T}}\right)\)
\(=2^{4} \times(\operatorname{det} \mathrm{A}) \times(\operatorname{det} \mathrm{B})^{-1} \times(\operatorname{det} \mathrm{C})^{3} \times\left(\operatorname{det} \mathrm{B}^{\mathrm{T}}\right)\)
\(=16 \times 5 \times(-3)^{-1} \times\left(\frac{1}{2}\right)^{3} \times(-3)\)
\(=\frac{80}{8}\)
\(=10\)
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5. If A is a $3 \times 3$ non-zero matrix such that $\mathrm{A}^{2}=0$ then determinant of $\left[(1+A)^{2}-50 A\right]$ is equal to

## Solutions:

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\(\because(\mathrm{x}+\mathrm{a})^{\mathrm{n}}={ }^{\mathrm{n}} \mathrm{C}_{0} \mathrm{x}^{\mathrm{n}} \mathrm{a}^{0}+{ }^{\mathrm{n}} \mathrm{C}_{1} \mathrm{x}^{\mathrm{n}-1} \cdot \mathrm{a}^{1}+{ }^{\mathrm{n}} \mathrm{C}_{2} \mathrm{x}^{\mathrm{n}-2} \cdot \mathrm{a}^{2}+\cdots \cdot+{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{n}} \mathrm{x}^{0} \mathrm{a}^{\mathrm{n}}\)
\((1+\mathrm{A})^{50}={ }^{50} \mathrm{C}_{0} 1^{50} \cdot \mathrm{~A}^{0}+{ }^{50} \mathrm{C}_{1} 1{ }^{49} \cdot \mathrm{~A}+{ }^{50} \mathrm{C}_{2} 1^{48} \cdot \mathrm{~A}^{2}+\cdots+{ }^{50} \mathrm{C}_{50} 1^{0} \mathrm{~A}^{50}\)
\(\because A^{2}=0\), then \(A^{3}=A^{4}=\cdots \cdot A^{50}=0\)
\(\therefore(1+\mathrm{A})^{50}={ }^{50} \mathrm{C}_{0} 1^{50} \mathrm{~A}^{0}+{ }^{50} \mathrm{C}_{1} 1^{49} \cdot \mathrm{~A}\)
\(=(1+50 \mathrm{~A})\)
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Now determinant $\left[(1+A)^{50}-50 \mathrm{~A}\right]$
$=$ Determinant $[1+50 \mathrm{~A}-50 \mathrm{~A}]$
= 1
6. Three friends divided some apples in the ratio $3: 5: 7$ among themselves. After consuming 16 apples they found that the remaining number of apples with them was equal to largest number of apples received by one of them at the beginning. Total number of apples these friends initially had was

## Solutions:

Let initially the 3 friends have $3 x, 5 x \& 7 x$ apples.
They had total 15 x apples.
According to the question,
$15 \mathrm{x}-16=7 \mathrm{x}$
$8 \mathrm{x}=16$
$\mathrm{x}=2$
Total apples $=15 \mathrm{x}$
$=15 \times 2=30$.
7. A shopkeeper reduces the price of a pen by $25 \%$ as a result of which the sales quantity increased by $20 \%$. If the revenue made by the shopkeeper decreases by $\mathrm{x} \%$, then x is...

## Solutions:

Revenue $=$ Price $\times$ Sales (in quantity)

$$
\downarrow ? ? \quad \downarrow 25 \% \quad \uparrow 20 \%
$$

Net $\%$ change in revenue $=\left(a+b+\frac{a b}{100}\right) \%$
$=\left[-25+20+\frac{(-25)(20)}{100}\right] \%$
$=[-5-5] \%$
$=-10 \%$
It means revenue will decrease by $10 \%$.
8. For all real values of $x, \frac{3 x^{2}-6 x+12}{x^{2}+2 x+4}$ lies between 1 and $k$, and does not take any value above $k$. Then $k$ equals $\qquad$

## Solutions:

$$
\begin{aligned}
& \text { let }=\frac{3 x^{2}-6 x+12}{x^{2}+2 x+4}=m \\
& x^{2}(3-m)-x(6+2 m)+12-4 m=0 \\
& \text { As } x \text { as real, } D \geq 0 \\
& \text { So, }(6+2 m)^{2}-4(3-m)(12-4 m) \geq 0 \\
& 36+24 m+4 m^{2}-\left[(12-4 m)^{2}\right] \geq 0 \\
& 36+24 m+4 m^{2}-\left[144+10 m^{2}-96 m\right] \geq 0 \\
& -108+120 m-12 m^{2} \geq 0 \\
& 12 m^{2}-120 m+108 \leq 0 \\
& m^{2}-10 m+9 \leq 0 \\
& m^{2}-9 m-m+9 \leq 0 \\
& m(m-9)-(m-9) \leq 0 \\
& (m-1)(m-9) \leq 0 \\
& -1 \leq m \leq 9 \\
& \text { So } k=9
\end{aligned}
$$

9. The maximum distance between the point $(-5,0)$ and a point on the circle $X^{2}+Y^{2}=4$ is......

## Solutions:



By observation,
Maximum distance of point $M(-5,0)$ from a point $Q(2,0)$ on the circle is 7 units.
10. If $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are positive real numbers such that $\mathrm{x}^{12}=\mathrm{y}^{16}=\mathrm{z}^{24}$, and the three quantities $3 \log _{\mathrm{y}} \mathrm{x}, 4 \log _{\mathrm{z}} \mathrm{y}, \mathrm{n} \log _{\mathrm{x}} \mathrm{z}$ are in arithmetic progression, then the value of $n$ is

## Solutions:

Given that $=\frac{3 \log _{y} x+x \cdot \log _{x} z}{2}=4 \log _{z} y$
$3 \log _{y} x+x \cdot \log _{x} z=8 \log _{z} y$
Also $x^{12}=y^{16}=z^{24}$
We take $\mathrm{x}^{12}=\mathrm{y}^{16}$
Taking $\log$ both sides, we get $\log x^{12}=\log y^{16}$

$$
\begin{aligned}
& 12 \log x=16 \log y \\
& \log _{y} x=\frac{16}{12}=\frac{4}{3}
\end{aligned}
$$

Similarly $\log _{z} y=\frac{3}{2}$ and $\log _{x} z=\frac{1}{2}$
Substituting these values in equation (i) we get
$3 \times \frac{4}{3}+\mathrm{n} \times \frac{1}{2}=8 \times \frac{3}{2}$
Solving we get $\mathrm{n}=16$
11. The number of pairs $(x, y)$ satisfying the equation $\sin x+\sin y=\sin (x+y)$ and $|x|+|y|=1$ is

## Solutions:

$|x|+|y|=1$
$\sin x+\sin y=\sin (x+y)$
Put, $x=0$ in (i) we get $y=1$
Put, $x=0$ in (i) we get $y=-1$
Put, $y=0$ in (i) we get $x=1$
Put, $y=0$ in (i) we get $x=-1$
Put, $x=\frac{1}{2}$ in (i) we get $y=\frac{-1}{2}$
Put, $x=\frac{-1}{2}$ in (i) we get $y=\frac{1}{2}$
All the alone value of $x \& y$ also satisfy the equation (ii)
Therefore number of pairs ( $\mathrm{x}, \mathrm{y}$ ) required $=6$
12. The $x^{2}+y^{2}-6 x-10 y+k=0$ does not touch or intersect the coordinate axes. If the point $(1,4)$ does not lie outside the circle, and the range of $k$ is $(a, b]$ then $a+b$ is $\qquad$

Solutions:
Standard from of circle $\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}=R^{2}$

$\Rightarrow \mathrm{x}^{2}+\mathrm{y}^{2}-6 \mathrm{x}-10 \mathrm{y}+\mathrm{k}=0$
$\Rightarrow\left(x^{2}-6 x+9\right)+\left(y^{2}-10 y+25\right)+\mathrm{k}-9-25=0$
$\Rightarrow(\mathrm{x}-3)^{2}+(\mathrm{y}-5)^{2}=34-\mathrm{k}$
Centre $=(3,5)$
Radius, $\mathrm{R}=\sqrt{34-\mathrm{k}}$ or $\mathrm{R}^{2}=34-\mathrm{k}$
It is given that, circle does not touch or intersect the coordinate axes,
It means ( $\mathrm{R}<3$ or $\mathrm{R}<5$ )
$\mathrm{R}<3$ or $\mathrm{R}^{2}<9$
Also, it is given that point $(1,4)$ does not lie outside the circle
It means, $\mathrm{PQ} \leq \mathrm{R}\left[\because \mathrm{PQ}=\sqrt{(3-1)^{2}+(5-4)^{2}}=\sqrt{5}\right]$
or $\mathrm{PQ}^{2} \leq \mathrm{R}^{2}$
$\mathrm{R}^{2} \geq 5$
From (i), (ii) \& (iii)
$5 \leq R^{2}<9$
$5 \leq 34-\mathrm{k}<9$
$25<\mathrm{k} \leq 29$
Then $\mathrm{a}+\mathrm{b}=25+29=54$
13. If a $3 \times 3$ matrix is filled with +1 ' $s$ and -1 ' $s$ such that the sum of each row and column of the matrix is 1 , then the absolute value of its determinant is...

## Solutions:

According to question,
Determinant $=\left|\begin{array}{ccc}1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1\end{array}\right|$
$=1(-1-1)-(-1)(1-1)+1(-1-1)$
$=-2+0-2$
$=-4$
Absolute value of determinant $=|-4|=4$
14. Let the set $=\{2,3,4, \ldots, 25\}$. For each $k \in P$, define $Q(k)=\{x \in P$ such that $x>k$ and $k$ divides $x\}$. Then the number of elements in the set $P-U_{k=2}^{25} Q(k)$ is

## Solutions:

$P=\{2,3,4, \ldots .25\}$
$\mathrm{Q}(\mathrm{k})=\{\mathrm{x} \in \mathrm{P}: \mathrm{x}>\mathrm{k} \& \mathrm{k}$ divides x$\}$
Number of elements in $\mathrm{P}-\mathrm{U}_{\mathrm{k}=2}^{25} \mathrm{Q}(\mathrm{k})$
So, first of all we have to find out
$Q(2) \cup Q(3) \cup Q(4) \cup \ldots \ldots \cup Q(25)$
$Q(2)=\{4,6,8,10,12,14,16,18,20,22,24\}$
$Q(3)=\{6,9,12,15,18,21,24\}$
$Q(4)=\{8,12,16,20,24\}$
$Q(5)=\{10,15,20,25\}$
$Q(6)=\{12,18,24\}$
$Q(7)=\{14,21\}$
$Q(8)=\{16,24\}$
$Q(9)=\{18$,
!
!
$Q(25)=\{ \}$
So we take the union of all above values and we notice that the numbers which are not present in any of the above set are prime numbers.
Therefore, $\mathrm{P}-\mathrm{U}_{\mathrm{K}=2}^{25} Q(k)=\{2,3,5,7,11,13,17,19,23\}$
15. The number of whole metallic tiles that can be produced by melting and recasting a circular metallic plate, if each of the tiles has a shape of a right-angled isosceles triangle and the circular plate has a radius equal in length to the longest side of the tile (Assume that the tiles and plate are of uniform thickness, and there is no loss of material in the melting and recasting process) is....

## Solutions:

Volume of circular plate $=\mathrm{n} \times$ volume of one metallic tile.
(let the radius of plate be 1 unit)
Area of plate $\times$ thickness $=\mathrm{n} \times$ Area of one metallic tile $\times$ thickness
$\pi(1)^{2}=\mathrm{n} \times \frac{1}{2} \times\left(\frac{1}{\sqrt{2}}\right)^{2}$
$4 . \pi=x$
$4 \times 3.14=\mathrm{x}$
$12.50=\mathrm{x}$
It means 12 tiles can be produced (integer)
16. If $|x|<100$ and $|y|<100$, then the number of integer solutions of $(x, y)$ satisfying the equation $4 x+7 y=3$ is

## Solutions:

$|\mathrm{x}|<100 \Rightarrow-100<\mathrm{x}<100$
$|y|<100 \Rightarrow-100<y<100$
Given, $\quad 4 \mathrm{x}+7 \mathrm{y}=3$
$\Rightarrow y=\frac{3-4 x}{7}$
Putting $x=6$ we get $y=-3$

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Putting $x=13$ we get $y=-7$
Putting $x=20$ we get $y=-11$

| $\vdots$ | $\vdots$ |
| :--- | :--- |
| $\vdots$ | $\vdots$ |

Putting $\mathrm{x}=97$ we get $\mathrm{y}=-55$
We can see the value of $x$ increases in the steps of 7 and $y$ decrease is in the steps of 4 .
So, the value of $x$ will reach +100 first. And its value will be $6,13,20, \ldots . . . ., 97$ (14 values)
And similarly the value of $x$ will reach -100 first. And its value will be $-1,-8,-15$, till....-99 (15 values)
Corresponding values of $y$ will be well within the range of -100 to 100 as they are increasing in the steps of 4 only.
Therefore, total solutions of equation $4 \mathrm{x}+7 \mathrm{y}=3$ will be 29 .
17. The average of five distinct integers is 110 and the smallest number among them is 100 . The maximum possible value of the largest integer is

## Solutions:

Let the 5 distinct integers be $a, b, c, d \& e$ in increasing order. Their average is 110.
Therefore their sum $=110 \times 5=550$
$\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}+\mathrm{e}=550$
$100+b+c+d+e=550$
$b+c+d+e=450$
In order to maximize ' e ', we need to minimize rest of them.
So let $b=101, \quad c=102 \& d=103$
$101+102+103+d=450$
$\mathrm{d}=450-306$
$\mathrm{d}=144$
18. Assume that all positive integers are written down consecutively from left to right as in 1234567891011......

The 6389th digit in this sequence is...

## Solutions:

1234567 $\qquad$
From 1-9 there are total 9 digits.
From $10-99$, there are total $(90 \times 2)=180$ digits
From 100-999, there are total $(900 \times 3)=2700$ digits
That means if we write from 1 to 999 ,
We would have used $(9+180+2700)$ digits i.e., 2889 digits
To reach 6389th digit, we still have to write ( $6389-2889=3500$ )digits after the last 3-digit number 999 . After 999 every number will consist of 4 digits.
As we have to write 3500 digit more after 999 to reach 6389 th digit,
That means we have to write
$\left(\frac{3500}{4}\right)=875$ four - digit numbers after 999.
Adding 875 to 999 we reach the last 4-digit number in the series which is 1874 .
So 4 is the digit at $6389^{\text {th }}$ position.
19. The number of pairs of integers whose sums are equal to their products is

Solutions:
$a+b=a b$
$\Rightarrow \mathrm{a}=\mathrm{b}(\mathrm{a}-1)$
$\Rightarrow b=a /(a-1)$
We observe that on RHS the numerator \& denominator are two consecutive integers.
Their ratio cannot be an integer except for the two cases where $\mathrm{a}=0$ or 2 .
So 2 solutions are possible for (a, b) i.e. ( 0,0 ) and (2, 2)
20. You have been asked to select a positive integer N which is less than 1000 , such that it is either a multiple of 4 , or a multiple of 6 , or an odd multiple of 9 . The number of such numbers is

## Solutions:

20. $m(4$ or 6$)=m(4)+m(6)-m(4 \& 8)$

$$
=m(4)+m(6)-m(12)
$$

$$
=[250-1]+[166]-[83]
$$

$$
\mathrm{m}[\mathrm{a}]=111 \text { [ } 56 \text { odd multiples }+55 \text { even multiples] }
$$

So $m(4)$ or $m(6)$ or odd $m(9)=249+166-83+56$

$$
=388
$$

As [No multiple will be common between $m$ (4) or $m(6)$ AND ODD $m(9)$ ]
21. If the compound interest earned on a certain sum for 2 years is twice the amount of simple interest for 2 years, then the rate of interest per annum is $\qquad$ percent
(a) $200 \%$
(b) $2 \%$
(c) $4 \%$
(d) $400 \%$

## Solutions:

Let the SI for 2 years be ₹ 200 \& CI be ₹ 400 .
We know SI for each year is equal.
Also the SI \& CI for $1^{\text {st }}$ year is equal.

|  | SI | CI |
| :--- | :--- | :--- |
| I year | 100 | 100 |
| II year | 100 | 300 |

From the table we can see that CI has increased from ₹100 in first year to ₹ 300 in $2^{\text {nd }}$ year.
$\%$ increase $=\frac{200}{100} \times 100=200 \%$, which is also the rate of interest.

## Alternate Solution

$\mathrm{CI}=2$. (SI) $\rightarrow$ In 2 years
$\mathrm{P}\left[\left(1+\frac{\mathrm{R}}{100}\right)^{2}-1\right]=2 . \frac{\mathrm{P} . \mathrm{R} \times 2}{100}$
Solving quadratic equation in $R$, we get $R=200 \%$
22. The maximum value of the natural number $n$ for which $21^{n}$ divides 50 ! is
(a) 6
(b) 7
(c) 8
(d) 9

## Solutions:

$21=7 \times 3$
So, one pair of $7 \& 3$ multiplies to produce one 21 .
We need to find highest power of 7 that completely divides 50 .
$\left[\frac{50}{7}\right]=7$
$\left[\frac{7}{7}\right]=1$
$=>7+1=8$.
We need not to find the highest power of 3 as it will be in any case more than power of 7 .
Thus, only 8 pairs of $7 \& 3$ can be obtained in factorized form in 50 !
So, we can say the maximum value of x for which $21^{\mathrm{x}}$ divides 50 is 8 .
23. The remainder when $\left(29^{29}\right)^{29}$ is divided by 9 is
(a) 1
(b) 2
(c) 3
(d) 4

## Solutions:

Rem $\left[\frac{\left(29^{29}\right)^{29}}{9}\right]$
$=\operatorname{Rem}\left[\frac{(27+2)^{29 \times 29}}{9}\right]=\operatorname{Rem}\left[\frac{(2)^{29 \times 29}}{9}\right]=\operatorname{Rem}\left[\frac{(2)^{841}}{9}\right]$
$=\operatorname{Rem}\left[\frac{(2)^{3 \times 280} \times 2}{9}\right]=\operatorname{Rem}\left[\frac{(8)^{280} \times 2}{9}\right]=\operatorname{Rem}\left[\frac{(9-1)^{280} \times 2}{9}\right]$
$=\operatorname{Rem}\left[\frac{(-1)^{280} \times 2}{9}\right]=\operatorname{Rem}\left[\frac{1 \times 2}{9}\right]=\operatorname{Rem}\left[\frac{2}{9}\right]$
So, the remainder is 2 .
24. Placing which of the following two digits at the right end of 4530 makes the resultant six digit number divisible by 6,7 and 9 ?
(a) 96
(b) 78
(c) 42
(d) 54

## Solutions:

In this question, we shall go with the option only \& apply the rule of divisibility of 6,7 and 9
Option (a)
We see by placing 96 at the right end of 4530 , the number becomes
453096, which is divisible by $6,7 \& 9$
$\therefore 96$ is the answer.
25. In a school $70 \%$ of the boys like cricket and $50 \%$ like football. If $x \%$ like both Cricket and Football, then
(a) $20 \leq x \leq 50$
(b) $\mathrm{x} \leq 20$
(c) $\mathrm{x} \geq 50$
(d) $10 \leq x \leq 70$

## Solutions:



In order to get maximum value of $n(C \cap F)$, we need to maximize $n(C \cup F)$.
Now the maximum value of $n(C \cup F)$ or the no. of boys who like at least one of the games is $100 \%$
So, $\mathrm{n}(\mathrm{C} \cap \mathrm{F})=\mathrm{x} \%=70 \%+50 \%-100 \%$
$\mathrm{x} \%=120 \%-100 \%$
$\mathrm{x} \%=20 \% \quad$ (Minimum value)
Also the diagram show that maximum value of x can be $50 \%$ if all boys playing football also plays cricket. Therefore $20 \leq \mathrm{x} \leq 50$.
26. In a class of 65 students 40 like cricket, 25 like football and 20 like hockey, 10 students like both cricket and football, 8 students like football and hockey and 5 students like all three sports. If all the students like at least one sport, then the number of students who like both cricket and hockey is
(a) 7
(b) 8
(c) 10
(d) 12

## Solutions:


$\mathrm{n}(\mathrm{C} \cup \mathrm{F} \cup \mathrm{H})=\mathrm{n}(\mathrm{C})+\mathrm{n}(\mathrm{F})+\mathrm{n}(\mathrm{H})-\mathrm{n}(\mathrm{C} \cap \mathrm{F})-\mathrm{n}(\mathrm{C} \cap \mathrm{H})-\mathrm{n}(\mathrm{F} \cap \mathrm{H})+\mathrm{n}(\mathrm{C} \cap \mathrm{F} \cap \mathrm{H})$
$65=40+25+20-10-\mathrm{n}(\mathrm{C} \cap \mathrm{H})-8+5$
$\mathrm{n}(\mathrm{C} \cap \mathrm{H})=40+25+20-10-8+5-65$
$n(C \cap H)=7$
27. If $x \in(a, b)$ satisfies the inequality, $\frac{x-3}{x^{2}+3 x+2}$ then the largest possible value of $b-a$ is
(a) 3
(b) 1
(c) 2
(d) No real values of x satisfies the inequality

## Solutions:

$\frac{x-3}{x^{2}+3 x+2} \geq 1$
$\Rightarrow \frac{x-3}{x^{2}+3 x+2}-1 \geq 0$
$\Rightarrow \frac{\mathrm{x}-3-\mathrm{x}^{2}-3 \mathrm{x}-2}{\mathrm{x}^{2}+3 \mathrm{x}+2} \geq 0$
$\Rightarrow \frac{-x^{2}-2 x-5}{x^{2}+3 x+2} \geq 0$
$\Rightarrow \frac{\mathrm{x}^{2}+2 \mathrm{x}+5}{\mathrm{x}^{2}+3 \mathrm{x}+2} \leq 0$
Discriminant (D) of $x^{2}+2 \mathrm{x}+5 \quad[\because \mathrm{a}=1, \mathrm{~b}=2, \mathrm{c}=5]$
$\mathrm{b}^{2}-4 \mathrm{ac}=(2)^{2}-4(1)(5)$
$=4-20<0$
If $\mathrm{D}<0 \& \mathrm{a}>0$ then $\mathrm{x}^{2}+2 \mathrm{x}+5>0$
For $\mathrm{x} \in(\mathrm{a}, \mathrm{b})$
If $\frac{x^{2}+2 x+5}{x^{2}+3 x+2} \leq 0 \& x^{2}+2 x+5>0$
Then $x^{2}+3 x+2<0$
$(x+1)(x+2)<0$
$-2<x<-1$
$\therefore \mathrm{b}-\mathrm{a}=(-1)-(-2)=-1+2=1$
28. If $a, b, c$ are real numbers $a^{2}+b^{2}+c^{2}=1$, then the set of values $a b+b c+c a c a n$ take is:
(a) $[-1,2]$
(b) $\left[-\frac{1}{2}, 2\right]$
(c) $[-1,1]$
(d) $\left[-\frac{1}{2}, 1\right]$

## Solutions:

$(a+b+c)^{2}=a^{2}+b^{2}+c^{2}+2(a b+b c+c a)$
$\because a, b, c$ are real, then $\left(a^{2}+b^{2}+c^{2}\right) \geq 0$
$\therefore a^{2}+b^{2}+c^{2}+2(a b+b c+c a) \geq 0$
$\mathrm{ab}+\mathrm{bc}+\mathrm{ca} \geq \frac{-1}{2}$
Also, $(a-b)^{2}+(b-c)^{2}+(c-a)^{2}=2\left[a^{2}+b^{2}+c^{2}-(a b+b c+c a)\right]$
$\because \mathrm{a}, \mathrm{b}, \mathrm{c}$ are real, then $(\mathrm{a}-\mathrm{b})^{2}+(\mathrm{b}-\mathrm{c})^{2}+(\mathrm{c}-\mathrm{a})^{2} \geq 0$
$\therefore 2\left[a^{2}+b^{2}+c^{2}-(a b+b c+c a)\right] \geq 0$
$1-(a b+b c+c a) \geq 0$
$(a b+b c+c a) \leq 1$
$\frac{-1}{2} \leq(a b+b c+c a) \leq 1$ or $\left[\frac{-1}{2}, 1\right]$
29. The inequality $\log _{2} \frac{3 x-1}{2-x}<1$ holds true for
(a) $x \in\left(\frac{1}{3}, 1\right)$
(b) $x \in\left(\frac{1}{3}, 2\right)$
(c) $x \in\left(0, \frac{1}{3}\right) \cup(1,2)$
(d) $x \in(-\infty, 1)$

Solutions:
$\log _{a} N$ is define $d$ if $N>0, a>0, a \neq 1$
$\log _{2}\left(\frac{3 x-1}{2-x}\right)<1$
$\frac{3 x-1}{2-x}>0$
It means,
$(2-x)(3 x-1)>0$
$(x-2)(3 x-1)<0$

$x \in\left(\frac{1}{3}, 2\right)$
If $\log _{\mathrm{a}} \mathrm{N}<1, a>1$, then $\mathrm{N}<a^{1}$
$\therefore \log _{2}\left(\frac{3 \mathrm{x}-1}{2-\mathrm{x}}\right)<1$
$\frac{3 x-1}{2-x}<2^{1}$
$\frac{3 x-1}{2-x}-2<0$
$\frac{3 x-1-4+2 x}{2-x}<0$
$\frac{5 x-5}{2-x}<0$
$\frac{x-1}{2-x}<0$

It means,
$(2-x)(x-1)<0$
$(x-2)(x-1)>0$

$x \in(-\infty, 1) \cup(2, \infty)$
From (i) \& (ii)
$x \in\left(\frac{1}{3}, 1\right)$
30. The set of values of $x$ which satisfy the inequality $0.7^{2 x^{2}-3 x+4}<0.343$ is
(a) $\left(\frac{1}{2}, 1\right)$
(b) $\left(\frac{1}{2}, \infty\right)$
(c) $\left(-\infty, \frac{1}{2}\right)$
(d) $\left(-\infty, \frac{1}{2}\right) \cup(1, \infty)$

## Solutions:

$(0.7)^{2 \mathrm{x}^{2}-3 \mathrm{x}+4}<(0.7)^{3}$
Applying rule of inequalities, we get $2 x^{2}-3 x+4>3$
(If $\mathrm{f}^{\mathrm{a}}<\mathrm{f}^{\mathrm{b}}$; where f is a postive fraction then $\mathrm{a}>\mathrm{b}$ )
$2 x^{2}-3 \mathrm{x}+1>0$
Solving above expression in the inequality as a quadratic equation we get 2 roots,
$\mathrm{x}=\frac{1}{2}$ or $\mathrm{x}=1$
Thus, the solution of above in inequality will be
$\left(-\infty, \frac{1}{2}\right) \cup(1,+\infty)$
31. A chord is drawn inside a circle, such that the length of the chord is equal to the radius of the circle. Now, two circles are drawn, one on each side of the chord, each touching the chord at its midpoint and the original circle. Let k be the ratio of the areas of the bigger inscribed circle and the smaller inscribed circle, then k equals
(a) $(2+\sqrt{3})$
(b) $(1+\sqrt{2})$
(c) $(7+4 \sqrt{3})$
(d) $(97+56 \sqrt{3})$

## Solutions:



Radius of bigger inscribed circle $=\left(\frac{a+\frac{\sqrt{3}}{2} a}{2}\right)$
Radius of smaller inscribed circle $=\left(a-\frac{\sqrt{3}}{2} a\right)$
Ratio of their area $=k=\frac{\pi\left[\left(a+\frac{\frac{\sqrt{3}}{2}}{2}\right)\right]^{2}}{\pi\left[\frac{a-\frac{\sqrt{3}}{2} a}{2}\right]^{2}}$
$=\frac{\left(1+\frac{\sqrt{3}}{2}\right)^{2}}{\left(1+\frac{\sqrt{3}}{2}\right)^{2}}$
$=\frac{(2+\sqrt{3})^{2}}{(2-\sqrt{3})^{2}}=\frac{4+3+4 \sqrt{3}}{4+3-4 \sqrt{3}}$
$=\frac{7+4 \sqrt{3}}{7-4 \sqrt{3}} \times \frac{7+4 \sqrt{3}}{7+4 \sqrt{3}}$
$=\frac{49+48+56 \sqrt{3}}{1}$
$=97+56 \sqrt{3}$
32. Points $P, Q, R$ and $S$ are taken on sides $A B, B C, C D$ and $D A$ of square $A B C D$ respectively, so that $A P: P B=B Q$ : $\mathrm{QC}=\mathrm{CR}: \mathrm{RD}=\mathrm{DS}: S A=1: \mathrm{n}$. Then the ratio of the area of PQRS to the area of ABCD is
(a) $1:(1+n)$
(b) $1: n$
(c) $1+\mathrm{n}^{2}:(1+\mathrm{n})^{2}$
(d) $(1+n):\left(1+n^{2}\right)$

Solutions:


The ratio of the area of $P Q R S$ to the area of $A B C D$ is
$=\frac{\text { Area sq ABCD }-4 \times \text { Area of } \triangle \text { DSR }}{\text { Area sq ABCD }}$
$=\frac{\left[(n+1)^{2}-\frac{n}{2} \times 4\right]}{(n+1)^{2}}$
$=\frac{\mathrm{n}^{2}+2 \mathrm{n}+1-2 \mathrm{n}}{(\mathrm{n}+1)^{2}}$
$=\frac{n^{2}+1}{(n+1)^{2}}$
33. On a circular path of radius 6 m a boy starts from a point A on the circumference and walks along a chord AB of length 3 m . He then walks along another chord BC of length 2 m to reach point C . The point B lies on the minor arc AC . The distance between point C and A is
(a) $\frac{\sqrt{15}+\sqrt{35}}{2} \mathrm{~m}$
(b) 8 m
(c) $\sqrt{ } 13 \mathrm{~m}$
(d) 6 m

## Solutions:



In $\triangle \mathrm{ABC}$
$\mathrm{AB}+\mathrm{BC}>\mathrm{AC} \quad[\because$ Sum of any two sides should be more than the third side $]$
$3+2>\mathrm{AC}$
$5>\mathrm{AC}$ or $\mathrm{AC}<5$
$\therefore$ Option (b) \& (a) are eliminated.
Also $\triangle \mathrm{ABC}$ is an obtuse angled triangle, where $\mathrm{AC}^{2}>\mathrm{AB}^{2}+\mathrm{BC}^{2}$

Option (c) says $A C=\sqrt{13}$ which does not satisfies above inequality. Therefore option (c) is also ruled out. Therefore option (a) is correct.
34. The area enclosed by the curve $2|x|+3|y|=6$ is
(a) 12 square units
(b) 3 square units
(c) 4 square units
(d) 24 square units

## Solutions:

The equation $2|x|+3|y|=6$ represents four equations for different range sets of $x \& y$.
(i) $2 x+3 y=6$ (when $x \& y$ both are $+v e)$
(ii) $-2 x+3 y=6$ (when $x$ is - ve $\& y$ is $+v e)$
(iii) $2 x-3 y=6$ (when $x$ is $+v e \& y$ is $-v e$ )
(iv) $-2 x-3 y=6$ (when $x \& y$ both are - ve)

$$
2 x+3 y=6
$$

Area $=4 \times$ area of $\triangle \mathrm{POQ}$
$=4 \times \frac{1}{2} \times 2 \times 3$
Area $=12$ sq. units
35. Two points on a ground are 1 m apart. If a cow moves in the field in such a way that it's distance from the two points is always in ratio 3 : 2 then
(a) the cow moves in a straight line
(b) the cow moves in a circle
(c) the cow moves in a parabola
(d) the cow moves in a hyperbola

## Solutions:

Let the two point be A and B with coordinates $(0,0) \&(1,0)$
Let $P(h, k)$ be the locus of a point of movement of cow.


As per question,
$\frac{\mathrm{AP}}{\mathrm{PB}}=\frac{3}{2}$
$\frac{\sqrt{(\mathrm{h}-0)^{2}+(\mathrm{k}-0)^{2}}}{(\mathrm{~h}-1)^{2}+(\mathrm{k}-0)^{2}}=\frac{3}{2}$
$\frac{\mathrm{h}^{2}+\mathrm{k}^{2}}{\mathrm{~h}^{2}+1-2 \mathrm{~h}+\mathrm{k}^{2}}=\frac{9}{4}$
$4 h^{2}+4 \mathrm{k}^{2}=9 \mathrm{~h}^{2}+9 \mathrm{k}^{2}-18 \mathrm{~h}+9$
$5 h^{2}+5 k^{2}-9 h+9=0$
It is equation of a circle
36. Given that $\cos x+\cos y=1$, the range of $\sin x-\sin y$ is
(a) $[-1,1]$
(b) $[-2,2]$
(c) $[0, \sqrt{3}]$
(d) $[-\sqrt{3}, \sqrt{3}]$

Solutions:
Given that $\cos x+\cos y=1$
Let $\sin x-\sin y=k$
$(\mathrm{i})^{2}+(\mathrm{ii})^{2}$
$\Rightarrow \cos ^{2} \mathrm{x}+\cos ^{2} \mathrm{y}+2 \cos \mathrm{x} \cos \mathrm{y}+\sin ^{2} \mathrm{x}+\sin ^{2} \mathrm{y}-2 \sin \mathrm{x} \sin \mathrm{y}=1+\mathrm{k}^{2}$
$\Rightarrow\left(\cos ^{2}+\sin ^{2} x\right)+\left(\cos ^{2} y+\sin ^{2} y\right)+2[\cos x \cos y-\sin x \sin y]=1+k^{2}$
$\Rightarrow 1+1+2 \cos (\mathrm{x}+\mathrm{y})=1+\mathrm{k}^{2}$
$\Rightarrow 2 \cos (\mathrm{x}+\mathrm{y})=\mathrm{k}^{2}-1$
$\Rightarrow \cos (\mathrm{x}+\mathrm{y})=\frac{\mathrm{k}^{2}-1}{2}$
$\because-1 \leq \cos \theta \leq 1$
$\therefore-1 \leq \frac{\mathrm{k}^{2}-1}{2} \leq 1$
$-2 \leq \mathrm{k}^{2}-1 \leq 2$
$-1 \leq \mathrm{k}^{2} \leq 3$
$0 \leq k^{2} \leq 3 \quad\left[\mathrm{k}^{2}=+\mathrm{ve}\right]$
$-\sqrt{3} \leq k \leq \sqrt{3}$
or $[-\sqrt{3}, \sqrt{3}]$
37. If $\sin \theta+\cos \theta=m$, then $\sin ^{6} \theta+\cos ^{6} \theta$ equals
(a) $\frac{3\left(\mathrm{~m}^{2}+1\right)}{4}$
(b) $\frac{3\left(\mathrm{~m}^{2}-1\right)}{4}$
(c) $1-\frac{3\left(\mathrm{~m}^{2}-1\right)}{4}$
(d) $1-\frac{3\left(\mathrm{~m}^{2}-1\right)^{2}}{4}$

## Solutions:

$\because \sin ^{6} \theta+\cos ^{6} \theta=1-3 \sin ^{2} \theta \cos ^{2} \theta$
It is given that $\sin \theta+\cos \theta=m$
Squaring on both side
$(\sin \theta+\cos \theta)^{2}=m^{2}$
$\sin ^{2} \theta+\cos ^{2} \theta+2 \sin \theta \cos \theta=m^{2}$
$1+2 \sin \theta \cos \theta=\mathrm{m}^{2}$
$\sin \theta \cos \theta=\frac{\mathrm{m}^{2}-1}{2}$
$\sin ^{2} \theta \cos ^{2} \theta=\left(\frac{\mathrm{m}^{2}-1}{2}\right)^{2}=\frac{\left(\mathrm{m}^{2}-1\right)^{2}}{4}$
$\therefore \sin ^{6} \theta+\cos ^{6} \theta=1-3 \sin ^{2} \theta \cos ^{2} \theta$
$=1-\frac{3\left(\mathrm{~m}^{2}-1\right)^{2}}{4}$
38. If inverse of the matrix $\left[\begin{array}{cc}2 & -0.5 \\ -1 & x\end{array}\right]$ is $\left[\begin{array}{ll}1 & 1 \\ 2 & 4\end{array}\right]$, then the value of $x$ is
(a) 0.5
(b) 1
(c) 2
(d) 3

## Solutions:

$\because$ A. $\mathrm{A}^{-1}=\mathrm{I}$
$\left[\begin{array}{cc}2 & -0.5 \\ -1 & \mathrm{x}\end{array}\right] \cdot\left[\begin{array}{ll}1 & 1 \\ 2 & 4\end{array}\right]=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
$\left[\begin{array}{cc}1 & 0 \\ -1+2 \mathrm{x} & -1+4 \mathrm{x}\end{array}\right]=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
$-1+2 \mathrm{x}=0 \quad$ or $-1+4 \mathrm{x}=1$
$2 \mathrm{x}=1 \quad$ or $4 \mathrm{x}=2$
$\mathrm{x}=\frac{1}{2}$ or $\mathrm{x}=\frac{2}{4}=\frac{1}{2}$
$\therefore \mathrm{x}=\frac{1}{2}$
39. The function $f(x)=\frac{x^{3}+5 x^{2}-8 x}{3}$ is
(a) positive and monotonically increasing for $\mathrm{x} \in\left(-\infty, \frac{5-\sqrt{57}}{2}\right)$ and $\mathrm{x} \in\left(\frac{5+\sqrt{57}}{2},+\infty\right)$
(b) negative and monotonically decreasing for $\mathrm{x} \in\left(-\infty, \frac{5-\sqrt{57}}{2}\right.$ and $\mathrm{x} \in\left(\frac{5+\sqrt{57}}{2},+\infty\right)$
(c) negative and monotonically increasing for $\mathrm{x} \in\left(-\infty, \frac{5-\sqrt{57}}{2}\right)$ and positive and monotonically increasing for $x \in\left(\frac{5+\sqrt{57}}{2},+\infty\right)$
(d) positive and monotonically increasing for $\mathrm{x} \in\left(-\infty, \frac{5-\sqrt{57}}{2}\right)$ and negative and monotonically decreasing for $x \in\left(\frac{5+\sqrt{57}}{2},+\infty\right)$

## Solutions:

$f(x)=\frac{x^{3}-5 x^{2}-8 x}{3}$
$f(x)=\frac{x\left(x^{2}-5 x-8\right)}{3}$
Now, $x^{2}-5 x-8=0$
$x=\frac{-(-5) \pm \sqrt{(-5)^{2}-4(1)(-8)}}{2(1)}$
$x=\frac{5 \pm \sqrt{57}}{2}$
$x=\frac{5+\sqrt{57}}{2}$ or $\frac{5-\sqrt{57}}{2}$
$\therefore \mathrm{x}^{2}-5 \mathrm{x}-8=0$
$\left[x-\left(\frac{5+\sqrt{57}}{2}\right)\right]\left[x-\left(\frac{5-\sqrt{57}}{2}\right)\right]=0$
$\mathrm{f}(\mathrm{x})=\frac{1}{3}(\mathrm{x})\left[\mathrm{x}-\left(\frac{5+\sqrt{57}}{2}\right)\right]\left[\mathrm{x}-\left(\frac{5-\sqrt{57}}{2}\right)\right]$

40. For $\mathrm{a}>\mathrm{b}>\mathrm{c}>0$, the minimum value of the function $f(\mathrm{x})=|\mathrm{x}-\mathrm{a}|+|\mathrm{x}-\mathrm{b}|+|\mathrm{x}-\mathrm{c}|$ is
(a) $2 a-b-c$
(b) $a+b-2 c$
(c) $a+b+c$
(d) a-c

## Solutions:

The minimum value of $f(x)$ will come at $b$.
Putting $x=b$ in the function, we get
$f(b)=|b-a|+|b-b|+|b-c|$
$=|-(a-b)|+0+|b-c|$
$=\mathrm{a}-\mathrm{b}+\mathrm{b}-\mathrm{c} \quad(\because \mathrm{a}>\mathrm{b}$ and $\mathrm{b}>\mathrm{c})$
$=\mathrm{a}-\mathrm{c}$
41. Let $\alpha, \beta$ be the roots of $x^{2}-x+p=0$ and $\gamma, \delta$ be the roots of $x^{2}-4 x+q=0$ where $p$ and $q$ are integers. If $\alpha, \beta$, $\gamma, \delta$ are in geometric progression then $\mathrm{p}+\mathrm{q}$ is
(a) -34
(b) 30
(c) 26
(d) -38

## Solutions:

$\alpha, \beta$ are the root of $x^{2}-x+p=0$
$\gamma, \delta$ are the root of $\mathrm{x}^{2}-4 \mathrm{x}+\mathrm{q}=0$
$\alpha+\beta=\frac{-(-1)}{1}=1$
$\alpha \beta=\frac{p}{1}=p$
$\gamma+\delta=\frac{-(-4)}{1}=4$
$(\gamma)(\delta)=\frac{\mathrm{q}}{1}=\mathrm{q}$
$\alpha, \beta, \gamma$ and $\delta$ are in G.P
let $\alpha, \beta, \gamma, \delta$ bea, ar, ar $^{2}$ and $a r^{3}$
$\because \alpha+\beta=1$
$a+a . r=1$
$a(1+r)=1$
$\gamma+\delta=4$
$\mathrm{ar}^{2}+\mathrm{ar}^{3}=4$
$\operatorname{ar}^{2}(1+r)=4$

Now dividing (ii)by (i) we get
$\frac{\mathrm{ar}^{2}(1+\mathrm{r})}{\mathrm{a}(1+\mathrm{r})}=\frac{4}{1}$
$\mathrm{r}^{2}=4$
$r= \pm 2 \quad[\because \mathrm{p} \& \mathrm{q}$ are integer $\therefore$ neglect $\mathrm{r}=2]$
$\mathrm{a}(1+r)=1$
Put $r=-2, \quad \mathrm{a}(1-2)=1$
a $(-1)=1$
$\mathrm{a}=-1$
$\alpha \beta=p$
a $\times$ ar $=p$
$\mathrm{a}^{2} \mathrm{r}=\mathrm{p}$
$(-1)^{2}(-2)=\mathrm{p}$
$p=-2$
$\& \gamma \delta=q$
$\mathrm{ar}^{2} \times \mathrm{ar}^{3}=\mathrm{q}$
$a^{2} r^{5}=\mathrm{q}$
$(-1)^{2}(-2)^{5}=\mathrm{q}$
$-32=q$
$\therefore p+q=(-2)+(-32)=-34$
42. If $\left(1+x-2 x^{2}\right)^{6}=A_{0}+\sum_{r=1}^{12} A_{r} X^{r}$, then value of $A_{2}+A_{4}+A_{6}+\cdots+A_{12}$ is
(a) 31
(b) 32
(c) 30
(d) 29

Solutions:
$\left(1+x-2 x^{2}\right)^{6}=1+A_{1} x+A_{2} x^{2}+A_{3} x^{3}+\cdots A_{12} x^{12}$
Put $\mathrm{x}=1$
$0=1+A_{1}+A_{2}+A_{3}+\cdots A_{12}$
Put $\mathrm{x}=-1$
$(1-1-2)^{6}=1-A_{1}+A_{2}-A_{3}+\cdots+A_{12}$
$64=1-A_{1}+A_{2}-A_{3}+\cdots+A_{12}$
Add equation (i) and (ii) we get
$64=2+2\left[A_{2}+A_{4}+\cdots A_{12}\right]$
$\frac{62}{2}=31=A_{2}+A_{4}+\cdots A_{12}$
43. The number of terms common to both the arithmetic progressions $2,5,8,11, \ldots ., 179$ and $3,5,7,9, \ldots . ., 101$ is
(a) 17
(b) 16
(c) 19
(d) 15

Solutions:
Common difference in $1^{\text {st }} \mathrm{AP}=3$
Common difference in $2^{\text {nd }} \mathrm{AP}=2$
Take LCM of 3 \& $2=6$
Now, we see that $1^{\text {st }}$ common term in both the series is 5 .
Next common term will come by adding 6 to it i.e. 11 and so on.

Last common term should be less than or equal to $101 \&$ it should be in the form $5+6 \mathrm{n}$, where $\mathrm{n}=0,1,2,3 \ldots \ldots 16$
So there are 17 common term in given two progressions.
44. From a pack of 52 cards, we draw one by one, without replacement. If $f(n)$ is the probability that an Ace will appear at the $n^{\text {th }}$ turn, then
(a) $\mathrm{f}(2)=\frac{1}{13}>\mathrm{f}(3)$
(b) $\frac{1}{13}>f(2)>f(3)$
(c) $f(3)>f(2)=\frac{1}{13}$
(d) $f(2)=f(3)=\frac{1}{13}$

## Solutions:

$f(2) \Rightarrow$ Probability that an ace will appear at 2 nd turn

$$
=\frac{48}{52} \times \frac{4}{51}=0.072
$$

$\mathrm{f}(3)=$ probabiltity that on ace will appear at 3rd turn.

$$
=\frac{48}{58} \times \frac{47}{51} \times \frac{4}{50}=0.068
$$

$\& \frac{1}{13}=0.076$
$\therefore$ (b)option is correct $\frac{1}{13}>\mathrm{f}(2)>\mathrm{f}(3)$
45. A die is thrown three times and the sum of the three numbers is found to be 15 . The probability that the first throw was a four is
(a) $\frac{1}{6}$
(b) $\frac{1}{4}$
(c) $\frac{1}{5}$
(d) $\frac{1}{10}$

## Solutions:

Let event $B$ is getting sum as 15 (known) and event $A$ is getting first dice outcome as 4 .
Number of ways of getting sum 15 are
$\begin{array}{lll}D_{1} & D_{2} & D_{3}\end{array}$
$(5,5, \quad 5) \quad \Rightarrow 1$ case
$(5,4,6) \Rightarrow 3!=6$ cases
$(3,6,6) \quad \Rightarrow 3$ cases
So $P(A / B)=\frac{P(A \cap B)}{P(B)}=\frac{2 / 216}{10 / 216}$
$=\frac{1}{5}$
46. In a given village there are only three sizes of families: families with 2 members, families with 4 members and families with 6 members. The proportion of families with 2,4 and 6 members are roughly equal. A poll is conducted in this village wherein a person is chosen at random and asked about his/her family size. The average family size computed by sampling 1000 such persons from the village would be closest to
(a) 4
(b) 4.667
(c) 4.333
(d) 3.667

## Solutions:

Given : Proportion of families with $2,4 \& 6$ member are equal.
So lets say there an 3 families, 1 family of each type.
That mean total no. of member will be
$2+4+6=12$ members.
If a sample of 12 member is taken,
then average size of family $=2 \times\left(\frac{2}{12}\right)+4 \times\left(\frac{4}{12}\right)+6 \times\left(\frac{6}{12}\right)$

$$
\begin{aligned}
& =\frac{1}{3}+\frac{4}{3}+3 \\
& =\frac{1+4+9}{3}=4.66
\end{aligned}
$$

Note: 1000 persons must have men from 2,4 and 6 member families in the ratio $2: 4: 6$ only, so the average will come same as 4.66
47. The value of $\left(\log _{3} 30\right)^{-1}+\left(\log _{4} 900\right)^{-1}+\left(\log _{5} 30\right)^{-1}$ is
(a) 0.5
(b) 30
(c) 2
(d) 1

## Solutions:

$$
\begin{aligned}
& =\frac{1}{\log 30}+\frac{1}{\log _{4} 900}+\frac{1}{\log _{5} 30} \\
& =\frac{\log 3}{\log 30}+\frac{\log 4}{2 \log 30}+\frac{\log 5}{\log 30}\left(\because \log _{b} a=\frac{\log a}{\log b} \text { and } \log a^{n}=n \log a\right) \\
& =\frac{1}{\log 30}\left(\log 3+\frac{1}{2} \log 4+\log 5\right) \\
& =\frac{1}{\log 30}(\log 3+\log 2+\log 5) \\
& =\frac{\log (3 \times 2 \times 5)}{\log 30} \\
& =\frac{\log 30}{\log 30} \\
& =1
\end{aligned} \quad(\because \log (\mathrm{ab})=\log a+\log b]
$$

Quantitative Ability (QA)
48. The inequality $\log _{a} \mathrm{f}(\mathrm{x})<\log _{a} \mathrm{~g}(\mathrm{x})$ implies that
(a) $\mathrm{f}(\mathrm{x})>\mathrm{g}(\mathrm{x})>0$ for $0<\mathrm{a}<1$ and $\mathrm{g}(\mathrm{x})>\mathrm{f}(\mathrm{x})>0$ for $\mathrm{a}>1$
(b) $\mathrm{g}(\mathrm{x})>\mathrm{f}(\mathrm{x})>0$ for $0<\mathrm{a}<1$ and $\mathrm{f}(\mathrm{x})>\mathrm{g}(\mathrm{x})>0$ for $\mathrm{a}>1$
(c) $\mathrm{f}(\mathrm{x})>\mathrm{g}(\mathrm{x})>0$ for $\mathrm{a}>0$
(d) $\mathrm{g}(\mathrm{x})>\mathrm{f}(\mathrm{x})>0$ for $\mathrm{a}>0$

## Solutions:

$\log _{a} N$ is defined if $N>0, a>0 \& a \neq 1$
$\log _{\mathrm{a}} \mathrm{f}(\mathrm{x})<\log _{\mathrm{a}} \mathrm{g}(\mathrm{x})$
If $0<a<1, f(x)>g(x)$
If $\mathrm{a}>1, \mathrm{f}(\mathrm{x})<\mathrm{g}(\mathrm{x})$
So, $\mathrm{f}(\mathrm{x})>\mathrm{g}(\mathrm{x})>0$ for $0<\mathrm{a}<1$
$\& \mathrm{~g}(\mathrm{x})>\mathrm{f}(\mathrm{x})>0$ for $\mathrm{a}>1$
49. Three cubes with integer edge lengths are given. It is known that the sum of their surface areas is $564 \mathrm{~cm}^{2}$ Then the possible values of the sum of their volumes are $\qquad$
(a) $764 \mathrm{~cm}^{3}$ and $586 \mathrm{~cm}^{3}$
(b) $586 \mathrm{~cm}^{3}$ and $564 \mathrm{~cm}^{3}$
(c) $764 \mathrm{~cm}^{3}$ and $564 \mathrm{~cm}^{3}$
(d) $586 \mathrm{~cm}^{3}$ and $786 \mathrm{~cm}^{3}$

## Solutions:

Let the edge of 3 cubes be $a, b$ and $c$
So, $6 \mathrm{a}^{2}+6 \mathrm{~b}^{2}+6 \mathrm{c}^{2}=564$
$6\left(\mathrm{a}^{2}+\mathrm{b}^{2}+\mathrm{c}^{2}\right)=564$
$a^{2}+b^{2}+c^{2}=94$
There could be 2 sets of ( $a, b, c$ ) using hit and trial method.
(i) $2,3,9$
(ii) 3, 6, 7 in any order,

So the possible values of the sum of their volumes are
(i) $2^{3}+3^{3}+9^{3}=764$
(ii) $3^{3}+6^{3}+7^{3}=586$.
50. Determine the greatest number among the following four numbers
(a) $2^{300}$
(b) $3^{200}$
(c) $2^{100}+3^{100}$
(d) $4^{100}$

## Solutions:

We need to find the greatest number.
Take option (d)
$4^{100}=2^{200}$
So it is less than option (a) which is $2^{300}$
So, option (d) is eliminated.
Now, take option (c)

$$
\begin{aligned}
2^{100}+3^{100} & <3^{100}+300^{100} \\
& <2.3^{100} \\
& <3^{100} \cdot 3^{100} \\
& <3^{200} \\
& <\text { option(b) }
\end{aligned}
$$

So option (c) is also eliminated.

Finally option (a) \& option (b) to be compared.

| Option (a) <br> $2^{300}$ <br> $=2^{100} \cdot 2^{200}$ <br> $=2^{100}$ | option (b) <br> $3^{200}$ <br> $=3^{200}$ <br> $=3^{200}$ <br> $2^{200}$$\quad$ (dividing both sides by |
| :--- | :--- |
| $\left.2^{2000}\right)$ |  |
|  | $=\left(\frac{3}{2}\right)^{200}=(1.5)^{200}=(2.25)^{100}$ |

As $2^{100}<(2.25)^{100}$ we can see L.H.S $<$ R.H.S
$\therefore 2^{300}<3^{200}$
From above we can say option (a) lesser than option (b)
So, option (a) is eliminated option (b) is highest.
51. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0,0),(0,31)$, and $(31,0)$ is
(a) 435
(b) 465
(c) 450
(d) 464

## Solutions:



So adding $1+2+3+\ldots . . . . . .28+29$, we get
$=\frac{29 \times 30}{2}$
$=15 \times 29$
$=435$
52. Two small insects, which are x metres apart, take u minutes to pass each other when they are flying towards each other, and $v$ minutes to meet each other when they are flying in the same direction. Then, the ratio of the speed of the slower insect to that of the faster insect is
(a) $\frac{u}{v}$
(b) $\frac{u}{v-u}$
(c) $\frac{v-u}{v+u}$
(d) $\frac{u}{v+u}$

## Solutions:

Let speed of faster insect $=\mathrm{a} \mathrm{m} / \mathrm{min}$
Speed of slower insect $=\mathrm{bm} / \mathrm{min}$
According to the question,
$u=\frac{x}{a_{x}+b}$
$v=\frac{a-b}{a-b}$
Dividing (ii) by (i) we get
$\frac{\mathrm{a}+\mathrm{b}}{\mathrm{a}-\mathrm{b}}=\frac{v}{u}$
$\frac{\mathrm{a}}{\mathrm{b}}=\frac{\mathrm{v}+\mathrm{u}}{\mathrm{v}-\mathrm{u}}$
$\therefore \frac{\mathrm{b}}{\mathrm{a}}=\frac{\mathrm{v}-\mathrm{u}}{\mathrm{v}+\mathrm{u}}$
53. An alloy $P$ has copper and zinc in the proportion of 5: 2 (by weight), while another alloy $Q$ has the same metals in the proportion of $3: 4$ (by weight). If these two alloys are mixed in the proportion of $\mathrm{a}: \mathrm{b}$ (by weight), a new alloy $R$ is formed, which has equal contents of copper and zinc. Then, the proportion of copper and zinc in the alloy S , formed by mixing the two alloys P and Q in the proportion of $\mathrm{b}: \mathrm{a}$ (by weight) is
(a) $7: 9$
(b) $9: 7$
(c) $9: 5$
(d) $5: 9$

## Solutions:

Using alligation method,


It mean $\mathrm{a}: \mathrm{b}=1: 3$
As per the question, if we add $P \& Q$ in the ratio $b:$ a which means in the ratio $3: 1$, we well obtain $S$.

Again using alligation method,


Above ratio should be equal to $3: 1$
Equating them we set,
$\frac{\frac{3}{7}-x}{x-\frac{5}{7}}=\frac{3}{1}$
$\frac{3-7 x}{7 x-5}=\frac{3}{1}$
$3-7 x=21 x-15$
$18=28 x$
$\frac{9}{14}=x$, where $x$ is the fraction of copper in alloy $S$
So ratio of copper and zinc in alloy $S=9: 5$
54. How many different numbers can be formed by using only the digits 1 and 3 which are smaller than 3000000 ?
(a) 64
(b) 128
(c) 190
(d) 254

## Solutions:

Number of 7 digit nos.
$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2=1 \times 2^{6}$
( $1^{\text {st }}$ digit can only be 1 as we have to form nos. less than 30 lakhs)
Number of 6 digit nos.
$2 \times 2 \times 2 \times 2 \times 2 \times 2=2^{6}$
Number of 5 digit nos.
$2 \times 2 \times 2 \times 2 \times 2=2^{5}$
:
:
And number of single digit nos. $=2^{1}$
Adding all we $\left(2^{1}+2^{2}+2^{3}+2^{4}+2^{5}+2^{6}\right)+2^{6}$
$=190$
55. There are numbers $\mathrm{a}_{1}, \mathrm{a}_{2}, \mathrm{a}_{3}, \ldots$, , an each of them being +1 or -1 . If it is known that $a_{1} a_{2}+a_{2} a_{3}+a_{3} a_{4}+$ $\cdots a_{n-1} a_{n}+a_{n} a_{1}=0$ then
(a) $n$ is a multiple of 2 but not a multiple of 4
(b) $n$ is a multiple of 3
(c) n can be any multiple of 4
(d) The only possible value of $n$ is 4

## Solutions:

It is known that $a_{1} a_{2}+a_{2} a_{3}+a_{3} a_{4}+\cdots a_{n-1} a_{n}+a_{n} a_{1}=0$
Given that each of the term can be +1 or -1
Going with the
Option (a) take $n=2$
$a_{1} a_{2}+a_{2} a_{1}=0$
(i) $\quad(1)(1)+(1)(1) \neq 0$
(ii) $1(-1)+(-1)(1) \neq 0$
(iii) $\quad(-1)(1)+(1)(-1) \neq 0$
(iv) $(-1)(-1)+(-1)(-1) \neq 0$

In none of the above cases expression equals to 0 . So eliminated.

## Option (b)

Take $n=3$
$\mathrm{a}_{1} \mathrm{a}_{2}+\mathrm{a}_{2} \mathrm{a}_{3}+\mathrm{a}_{3} \mathrm{a}_{1}=0$
(i) $(1)(-1)+(-1)(-1)+(-1) 1 \neq 0$
(ii) $(1)(-1)+(-1)(1)+(1)(1) \neq 0$
(iii) $(1)(1)+(1)(-1)+(-1)(1) \neq 0$
(iv) $(1)(1)+(1)(1)+(1)(1) \neq 0$
(v) $(-1)(1)+(1)(1)+(1)(-1) \neq 0$
(vi) $(-1)(-1)+(-1)(1)+(1)(-1) \neq 0$

In none of the above cases expression equals to 0 . So eliminated.

## Option (c)

Take $\mathrm{n}=4$
$\mathrm{a}_{1} \mathrm{a}_{2}+\mathrm{a}_{2} \mathrm{a}_{3}+\mathrm{a}_{3} \mathrm{a}_{4}+\mathrm{a}_{4} \mathrm{a}_{1}=0$
(i) $(1)(-1)+(-1)(-1)+(-1)(1)+1 \times 1=0$.

Take $n=8$
$a_{1} a_{2}+a_{2} a_{3}+a_{3} a_{4}+a_{4} a_{5}+a_{5} a_{6}+a_{6} a_{7}+a_{7} a_{8}+a_{8} a_{1}=0$
(i) $(1)(-1)+(-1)(-1)+(-1)(1)+(1)(1)+(1)(-1)+(-1)(-1)+(-1)(1)+(1)(1)=0$

In each of the above cases expression equals to 0 .
Hence option (c) is correct.

Directions (Q.56-Q.60): Analyze the given data for exports and imports of rubber in Rs. crores from 2016 to 2017 and answer the questions based on the analysis.

Trade in Rubber (2006 to 2017)


- Exports

DImports
56. Average annual exports for the given period $2006-2017$ was approximately
(a) Rs. 230 Cr
(b) Rs. 220 Cr
(c) Rs. 210 Cr
(d) Rs. 190 Cr

## Solutions:

Average annual exports for the given period 2006-2017
$=\frac{280+280+230+210+200+220+210+200+200+200+220+200}{12}$
$=220$ (approx.)
57. The percentage decline in exports during the period 2006-2011 is more than the percentage decline in exports during 2012-2017 by approximately
(a) 16.5
(b) 20.5
(c) 12.5
(d) 21.5

Solutions:
$\%$ decline from $2006-2011$ in export $=\frac{220-280}{280} \times 100$
$=\frac{-60}{280}$
$=\frac{3}{14} \times 100$
$=21.9$
$\%$ decline in export from $2012-2017=\frac{190-200}{200} \times 100$
$=\frac{10}{200} \times 100$
= $5 \%$
So the difference is ( $21.9 \%-5 \%$ )
$=16.9=16.5 \%$ (approx.)

Quantitative Ability (QA)
58. The maximum difference between imports and exports is
(a) Rs. 60 Cr
(b) Rs. 110 Cr
(c) Rs. 120 Cr
(d) Rs. 100 Cr

## Solutions:

Maximum difference between import and exports is in 2014
$310-190=120 \mathrm{cr}$
59. Balance of trade is defined as imports subtracted from exports ( = exports - imports). Which of the following blocks of three years has witnessed the largest average negative balance of trade?
(a) 2007-2009
(b) 2015-2017
(c) 2014-2016
(d) 2010-2012

## Solutions:

Average balance of trade $=\frac{\text { Total imports }- \text { Total exports }}{\text { Number of years }}$
Option (a) 2007-2009
Average balance of trade $=\frac{(240+260+280)-(280+230+210)}{3}=20 \mathrm{Cr}$

Option (b) 2015-2017
Average balance of trade $=\frac{(310+250+240)-(200+220+200)}{3}=60 \mathrm{Cr}$

Option (c) 2014-2016
Average balance of trade $=\frac{(320+310+250)-(200+200+220)}{3}=120 \mathrm{Cr}$
Option (d) 2010-2012
Average balance of trade $=\frac{(310+290+320)-(220+210+200)}{3}=290 \mathrm{Cr}$
When imports are more than exports we say it is negative balance of trade.
The largest average negative balance of trade is 290 Cr . in 2010-2012
60. The percentage increase in imports over the previous year is maximum during
(a) 2009 to 2010
(b) 2010 to 2011
(c) 2013 to 2014
(d) 2008 to 2009

## Solutions:

Percentage increase in imports $=\frac{\text { (final value of imports }- \text { initial value of imports) }}{\text { initial value of imports }} \times 100$
Option (a) 2009-2010
$\%$ increase $=\frac{310-280}{280} \times 100=10.71 \%$

Option (b) 2010-2011

By Toprankers
$\%$ increase $=\frac{290-310}{310}=-6.45 \%$

Option (c) 2013-2014
$\%$ increase $=\frac{320-300}{300}=6.66 \%$

Option (d) 2008-2009
$\%$ increase $=\frac{280-260}{260}=7.69 \%$

So, the percentage increase in imports over the previous year is maximum during 2009-2010

By Toprankers

## IPMAT INDORE 2019

Passage (Q.61-Q.66): Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.

Supposing half a dozen or a dozen men were cast ashore from a wreck on an uninhabited island and left to their own resources, one of course, according to his capacity, would be set to one business and one to another; the strongest to dig and to cut wood, and to build huts for the rest: the most dexterous to make shoes out of bark and coats out of skins; the best educated to look for iron or lead in the rocks, and to plan the channels for the irrigation of the fields. But though their labours were thus naturally severed, that small group of shipwrecked men would understand well enough that the speediest progress was to be made by helping each other-not by opposing each other; and they would know that this help could only be properly given so long as they were frank and open in their relations, and the difficulties which each lay under properly explained to the rest. So that any appearance of secrecy or separateness in the actions of any of them would instantly, and justly, be looked upon with suspicion by the rest, as the sign of some selfish or foolish proceeding on the part of the individual. If, for instance, the scientific man were found to have gone out at night, unknown to the rest, to alter the sluices, the others would think, and in all probability rightly think, that he wanted to get the best supply of water to his own field; and if the shoemaker refused to show them where the bark grew which he made the sandals of, they would naturally think, and in all probability rightly think, that he didn't want them to see how much there was of it, and that he meant to ask from them more corn and potatoes in exchange for his sandals than the trouble of making them deserved. And thus, although each man would have a portion of time to himself in which he was allowed to do what he chose without let or inquiry - so long as he was working in that particular business which he had undertaken for the common benefit, any secrecy on his part would be immediately supposed to mean mischief; and would require to be accounted for, or put an end to: and this all the more because, whatever the work might be, certainly there would be difficulties about it which, when once they were well explained, might be more or less done away with by the help of the rest; so that assuredly every one of them would advance with his labour not only more happily, but more profitably and quickly, by having no secrets, and by frankly bestowing, and frankly receiving, such help as lay in his way to get or to give.
61. When a dozen men are cast away on an imaginary island, the best educated would look for metals in rocks because
(a) metals can be used to make weapons.
(b) such an island probably has unexploited resources.
(c) he may find it beneath him to dig or cut or make shoes.
(d) he is suited for such work.

Solution: option d. He's suited for such work. Each man in the island was given work according to his capacity. Refer to the second line of the first paragraph.
62. The author states that any appearance of secrecy or separateness would instantly and justly be looked upon with suspicion. From this statement we may infer that
(a) what is secret is not what is separate
(b) secrecy is not exactly the same as separateness
(c) it is natural to be suspicious of secrecy
(d) it only takes an instant for a relationship to deteriorate

Solution: option c. According to the given context, if any man on the uninhabited island acted secretively or tended to stay aloof, the others would naturally be suspicious.

By Toprankers
63. The instance of the shoemaker who refuses to show his source and asks for more corn and potatoes, is an example of
(a) a strong bargain.
(b) unfair practice.
(c) the system of barter.
(d) the intent to make trouble.

Solution: option b. Unfair practice. According to the author, if the shoemaker refused to show his source and asked for more corn, the others would think that he meant to ask from them more corn and potatoes in exchange for his sandals than the trouble of making them deserved.
64. According to the author, whatever one's work might be
(a) hardships are going to be part of it.
(b) one cannot keep complaining.
(c) one should expect others to assure of help and advance our labours.
(d) one must offer help to others in order to receive help.

Solution: Option a. Refer to the fourth last line of the passage. According to which, whatever the work might be, certainly there would be difficulties about it.
65. The author's belief is that for progress to happen
(a) a team should consist of people with multiple talents.
(b) co-operation among team members is essential.
(c) one must deal with those who are secretive.
(d) transparency among all concerned is mandatory.

Solution: option d. Refer to the phrase in the last line of the paragraph," by having no secrets"
66. The writer makes a hypothesis, which can be related to
(a) communities in general.
(b) an imaginary island, rich with resources.
(c) an ideal world of talented people.
(d) a primitive and unsophisticated world.

Solution: Option a. Communities in general. The passage doesn't talk about people with talents on an imaginary, primitive and unsophisticated island rich with resources. It talks about an uninhabited island where some shipwrecked men are given work according to their capacity.

Passage (Q.67-Q.72): Read the following passage and choose the answer that is closest to each of the questions that are based on the passage.
The perennial debate over gender differences threatens to remain inconclusive. Stereotypes pertaining to male superiority and female submissiveness could be traced to earlier ages where assigned roles were needed as survival measures. But, can we today see a swing away from these stereotypes, or have they established a stranglehold on our perceptions? In this gendered world, we continue to live with notions that one's gender determines one's skills and preferences, from toys and colours to career choices. So the girl child will be presented with a Barbie doll, while the boy child will receive a Lego set.
Does that mean that our brains are different? This myth has been exploded by a British professor of cognitive neuroimaging. Her research attempts to establish how these stereotypes mould our ideas of ourselves. She examines how science has been misinterpreted or misused to ask the wrong questions, instead of challenging the status quo. She urges us to move beyond a binary view of people's brains and instead to see these as highly individualised, profoundly adaptable, and full of unbounded potential.. In other words, modern neuroscientists have identified no decisive category-defining differences between the brains of men and women.

As a result of these findings, we owe it to ourselves to dump the myths and look at ourselves afresh. We need to recognise that the male and female brain debate is a distraction, besides being based on inaccuracies. It is possibly harmful too, because it can be used as a hook to justify saying there is no point in girls doing science because they do not have a science brain; or compelling boys to opt for science because their brains are shaped for that subject. It can also condemn boys for being emotional, as this is seen as a feminine trait. And, most dangerous of all, to proclaim that boys, not girls, are meant to lead.
67. The research of a British professor of cognitive neuroimaging has succeeded in establishing that
(a) the brains of men and women are alike.
(b) science needs to challenge the status quo.
(c) society must break away from attempts at stereotyping gender issues.
(d) the potential of a human brain is not directly linked to gender.

Solution: Option d. Refer to the $5^{\text {th }}$ line of the $2^{\text {nd }}$ paragraph, "Her conclusive findings establish that no brain differences can be found that are solely gender related."
68. By referring to the world as "gendered' the writer wants to convey that
(a) gender differences can be detected right from childhood.
(b) society continues to be fixated on gender stereotypes.
(c) one's gender is bound to determine one's abilities.
(d) the debate on gender differences will never be resolved.

Solution: Option b. Refer to the $5^{\text {th }}$ line of the second para, "In this gendered world, we continue to live with notions that one's gender determines one's skills and preferences, from toys and colours to career choices." This means that the society is still fixated on gender stereotypes.
69. One of the dangers in adopting a binary view of the human brain is that it can
(a) promote the notion of feminine and masculine traits.
(b) determine as well as limit academic choices.
(c) lead to the distortion and misinterpretation of scientific data.
(d) be used to encourage male dominance and superiority.

Solution: Option a. binary view means two views, masculine and feminine traits. Option b and c can be eliminated as the passage doesn't talk about determination or limitation of academic choices and misinterpretation of scientific data. It does refer to encouraging male dominance and superiority but that would only be only one side of the coin.
70. The writer of this passage wants to emphasise the need to
(a) use new insights provided by scientific research for a better understanding of human abilities.
(b) continuously debate issues of gender differences to achieve human progress.
(c) question the findings of scientific inquiry into the functioning of the human brain.
(d) accept gender differences as essential to the survival of the human species.

Solution: option a. Refer to the entire last paragraph where the writer suggest that we need to dump the myths and look at ourselves a fresh. Option b, c and d can be eliminated on the grounds that the passage doesn't metion debating gender differences, questioning the scientific enquiries or accepting gender differences explicitly or implicitly.
71. The writer of this passage wants to emphasise the need to
(a) use new insights provided by scientific research for a better understanding of human abilities.
(b) continuously debate issues of gender differences to achieve human progress.
(c) question the findings of scientific inquiry into the functioning of the human brain.

Verbal Ability (VA)
(d) accept gender differences as essential to the survival of the human species.

Solution: Option A.
72. The antonym for "unbounded' (Para 2) is
(a) imprisoned
(b) aggressive
(c) restricted
(d) fearful

Solution: Option c. Restricted
Directions (Q.73-Q.78): Complete the following sentences by choosing the most appropriate phrase from the options given below.
73. Although he is recovering from his illness, he has to follow certain diet restrictions. He cannot eat junk food. Please do not pity him and $\qquad$
(a) give him some snack
(b) cut him some slack
(c) be slack with his eating
(d) cut down the snacks

Solution: Option b. Cut him some slack which means allow someone some leeway in their conduct. The part before the blank asks not to pity therefore the continuation of the thought should not allowing the next course of action.
74. The problems may be difficult, but all you have to do is $\qquad$ as long as you can.
(a) hang in up there
(b) hang on there
(c) hang on to that
(d) hang in there

Solution: Option D.
hang in there - remain persistent and determined in difficult circumstances
75. The standards set by the examination board are so high that it would be difficult for poorly prepared students to $\qquad$
(a) pass most errors
(b) past muster
(c) get past most errors
(d) pass muster

Solution: Option D.
pass muster - Meet a required standard
76. After all the alliances and arithmetic, the party is likely to $\qquad$ a majority in the assembly election.
(a) scrape through
(b) scrape together
(c) tape together
(d) shape together

Verbal Ability (VA)
By Toprankers
Solution: Option b. scrape together means to collect and manage something/someone with great difficulty Scrape through - just barely succeed
Option c and d are grammatically wrong.
77. I'll have to $\qquad$ because I don't know how Sheila's parents are going to react to this offer.
(a) count on my fingers
(b) face the music
(c) break the ice
(d) play it by ear

Solution: Option d
count on my fingers - shows the number is surprisingly low
face the music - be confronted with the unpleasant consequences of one's actions.
break the ice - do or say something to get a conversation started
play it by ear - proceed instinctively
78. If you had been more alert, this golden opportunity would not have $\qquad$
(a) escaped your fingers
(b) slipped off
(c) escaped away
(d) slipped through your fingers

Solution: Option d. slipped through your fingers - Lose something as you didn't make an effort

Directions (Q.79-Q.84): In each of the following sentences the incorrect part of the sentence is underlined. Choose an alternative from the four given options so that the sentence is rendered correct.
79. The place where her father disappeared and the reason why he did are unknown to her
(a) her father disappeared and the reason why he did
(b) where her father disappeared and the reason he did
(c) her father disappeared and the reason he did
(d) where her father disappeared and the reason why he did

Solution: Option B. Both "reason" and "why" need not be present in the sentence together as they convey the same meaning.
80. Which team has the best record, yours or theirs?
(a) Which of the teams has the best record
(b) Which one of the teams has the best record
(c) Which one team has the better record
(d) Which team has the better record

Solution: Option D. If the choice is between two people, comparative degree of "good" should be used. Option C will be eliminated as Which one team" is grammatically incorrect.
81. After sensing a problem with the factory workers, the personnel officer demanded to know who the union leaders had contacted to conduct the petition drive.
(a) whom the union leaders had contacted
(b) who had the union leaders had contacted
(c) whom the union leaders have contacted
(d) who union leaders contacted

Verbal Ability (VA)
By Toprankers

Solution: Option A. Who should be replaced with Whom since it is acting as a object of the sentence. Option B and C can be eliminated due to the the presence of "had" twice and "have" in option C.
82. His wrongdoing was completely exposed, but not once he apologised for his actions
(a) not once did he apologise
(b) never he made an apology
(c) not once did he give apology
(d) not once he made an apology

Solution: Option A. The second clause of the sentence beginning with but not requires the helping verb "did" to make it grammatically correct. The other options to be eliminated as apology is neither made or given. One just apologises.
83. The Municipal Council can no longer wash its hands off its responsibilities.
(a) wash its hands from its
(b) wash their hands off their
(c) wash their hand of their
(d) wash its hands of its

Solution: Option b. Municipal Corporation is a group of individuals working together. Therefore, "wash their hands off their" is the only apt answer. The other alternates are grammatically wrong.
84. The letter states that one can avail the service offered by the company till June 20,2019
(a) avail the service on offer from
(b) avail the service on offer by
(c) avail of the service offered by
(d) avail of the service offered from

Solution: Option b. Avail of will be wrong grammatically therefore option cand d will be eliminated. Use of "by" with the doer of the action is more appropriate therefore option B is the most appropriate answer choice.
$\qquad$ Infrastructure, in the form of paved surfaces, disrupts water absorption and lowers water retention. This leads to disastrous levels of flooding which diminishes the biodiversity and impoverishes the people of the region. Land should be used mindfully to prevent water logging during heavy rains.
(a) Climate change is not the only cause for flooding.
(b) Flooding can happen after heavy or low rainfall.
(c) Infrastructure can actually cause a lot of trouble during flooding.
(d) Water retention is less important to prevent flooding.

Solution: option A. Option B can be eliminated as low rainfall can't be a reason for flooding. Option C talks about Infrastructure whereas the passage begins with Infrastructure as a new idea. Option D can be eliminated as it is suggesting water retention is a lesser means to prevent flooding. It can't be an introductory sentence.
86. Few look forward to old age and all that it brings in its wake - deteriorating health, loss of vigour, restricted mobility, increasing dependence on others, not to mention a sense of foreboding and anxiety. Yet, one has to learn to cope with the onset of old age. Firstly, it is imperative to prepare to accept old age in spite of the restrictions or limitations it imposes on one's mobility. Equally important is the need to adopt a positive attitude towards life. Above all, peace of mind, is the efficacious balm that brings equanimity to
one's life. We must resign ourselves to growing old, and in the process let us try to make life as fulfilling and meaningful as possible.
(a) The role of humour and fun are indispensable as these are the spice of life and guaranteed to bring cheer and bonhomie, besides keeping one's mind off life's grim realities.
(b) Owing to advances in medical science, we can now expect to live well beyond 90 years.
(c) Physical debility and stiffening body joints 'creaking' in protest may make mobility difficult - something one should learn to take in one's stride stoically.
(d) Turning nostalgic and recalling 'those good old days' when they were young and life was radically different from what it is today, help one accept old age.

Solution: Option A. The passage talks about coping with the onset of old age very positively. Option C and D can be eliminated as they refer to difficulties and accepting old age (which is slightly on the negative side). Option B doesn't really go with the flow of the thoughts, hence eliminated.
87. The Arab Spring is widely believed to have stemmed from dissatisfaction with the rule of local governments, though some have speculated that wide gaps in income levels may have had a hand as well. Issues such as political corruption, human rights violations, unemployment, and educated but dissatisfied youth may have been responsible as well.
(a) Thus, youth unrest was the main reason for the Arab Spring
(b) Some also cite the 2009-10 Iranian election protests as one of the reasons behind the Arab Spring
(c) The Arab Spring was due to the wide gap between the haves and the have-nots
(d) To sum up, the Arab Spring was a series of anti-government protests, uprisings, and armed rebellions across the Arab world

Solution: Option B. The last line has to be conclusive. Option C and D will be eliminated as the passage doesn't talk about haves and have nots, income inequalities, armed rebellions etc. Option A will be eliminated as youth unrest was not the only reason for the Arab Spring.
Besides, the tone of option B (Some also cite) also goes well with that of the rest of the passage (is widely believed, some have speculated etc)
88. One who is $\qquad$ gets on with his job in spite of obstacles, while the one who is $\qquad$ hardly shows any progress. The latter spends all his time $\qquad$ about his troubles.
(a) artful, doubtful, speaking
(b) assiduous, querulous, whining
(c) hardworking, dishonest, gossiping
(d) hot-headed, scared, crying Question

Solution: Option B. The passage demands a positive adjective in the first blank and a negative one in the second. Option a and c can be eliminated as speaking and gossiping won't fit in the last blank since one can't speak or gossip about one's troubles. Option D can be eliminated as a hotheaded (impulsive) person is unlikely to get on with his job inspite of obstacles.
Artful - clever or skilful, especially in a crafty or cunning way
Assiduous - showing great care and perseverance
Querulous - complaining in a rather petulant or whining manner
Whine - to complain
Hotheaded - impulsive
89. The $\qquad$ of multiculturalism, in times of war or economic $\qquad$ tribalism is what causes those in power to confine groups of people with different $\qquad$ into ghettos or in communes on the margins of their cities.
(a) rise, doldrums, ideologies
(b) tyranny, growth, habits
(c) antithesis, prosperity, persuasions
(d) opposite, distress, ethnicities

Solution: Option D. There can neither be growth nor prosperity in the times of war therefore option B and C will be eliminated. Similarly, multiculturalism can't rise in times of war, hence option A will also be eliminated. The answer will be, "The opposite of multiculturalism, in times of war or economic distress. Tribalism is what causes those in power to confine groups of people with different ethnicities into ghettos or in communes on the margins of their cities."
Doldrums - a state or period of stagnation or depression
Tyranny - an act or the pattern of harsh, cruel, and unfair control over other people
Ethnicity - belonging to the same social group
Antithesis - Converse
90. That the artiste went about systematically to get traditional $\qquad$ back into the mainstream $\qquad$ and a textile culture for dance is to be celebrated.
(a) practices, processed
(b) motifs, created
(c) totems, evolved
(d) stories, described

Solution: Option B. Processed and described won't fit in the second blank hence option a and d can be eliminated. Motif is a better fit than totem. Hence option B
Motifs - design with a pattern
Totems - Something that has spiritual significance
91. In response to my friend's request, I decided to write her a letter, which I hoped would be honest and practical, while also serving as a $\qquad$ of sorts for my own feminist thinking. This book is a $\qquad$ of that letter, with some details changed.
(a) map, version
(b) chart, form
(c) base, fallout
(d) guide, précis

Solution: Option A.
Fallout - adverse result of a situation
Precis - gist
Version - different form of the same thing
92. Quantum Physics really begins to point to this discovery. It says that you can't have a Universe without mind
$\qquad$ into $i t$, and that the mind is actually $\qquad$ the very thing that is being
(a) getting, creating, acknowledged
(b) intruding, making, construed
(c) entering, shaping, perceived
(d) penetrating, forming, seen

Solution: Option C. Option B and D can be eliminated as one's mind can't intrude or penetrate into the Universe. Similarly one can't acknowledge it, therefore Option A can also be eliminated.

Directions (Q.93-Q.97): One of the statements below contains a word used incorrectly. Choose the option which has the incorrect or inappropriate usage of the word.
93.
(a) The emperor ordered the arrest of his most vocal critic.
(b) There are very few film critics left in our city.
(c) Mahatma Gandhi's critic of the West remains relevant to this day.
(d) His classmates warned him, "It is not a good idea to critique our Principal."

Verbal Ability (VA)

## Solution: Option C

Critic ( n ) - a person who expresses an unfavourable opinion of something.
Critique (v) - evaluate
94.
(a) The river teemed with salmon and trout.
(b) The wedding was teamed in Bhojpuri style.
(c) Which team will win this year's IPL tournament?
(d) The film star wore a purple suit teamed with a crimson tie.

Solution: Option B
Teem - be full of or swarming with.
Team - a group of players forming one side in a competitive game or sport.
95.
(a) Last week we learnt about the right way to greet our customers.
(b) "Get it write the first time" is an often-heard management slogan.
(c) In India, driving on the Right side of the road is wrong.
(d) The rite of afternoon tea is described in many of Enid Blyton's books.

## Solution: Option B

Write (v)- to mark symbols, letters or words on a surface
Rite (n) - a ceremony
Right (adj)- denotes a direction or being correct
96.
(a) The attackers decided to raise the castle to the ground.
(b) They raised their children to be freethinking boys and girls.
(c) To raise a toast to a newly-wed couple is a common practice.
(d) The rays of the sun rose above the mountain.

Solution: Option A
Raise - Develop or nourish someone
Rays (n)- beam
Raze to the ground - To destroy completely
97.
(a) Shakespeare is sometimes referred to as a bard.
(b) He barred his soul to the preacher.
(c) Because of his age, he was barred from entering the theatre.
(d) The Bar barred all bards.

Solution: Option B
Barred - to be blocked from entrance or not allowed to do something
Bard - A poet who orates his work
Bared - uncovered
Directions (Q.98-Q.100): The sentences given below, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a number. Decide on the most logical order and enter the sequence of numbers in the space provided.
98.

1. He just harvested the wild grains.
2. The hunter-gatherer went from place to place in search of food.
3. As the crops began to give better yields, this reduced his need to go in search of animals and wild plants.
4. This was followed by an attempt to grow food by scattering the spare grains.

Solution: Sentence 2, "The hunter-gatherer went from place to place in search of food" looks like a broad introductory statement which can be followed sentences 413 seem to be triples as they talk about attempting to grow food by harvesting spare grains then harvesting them and finally crops giving better yield. Hence 2413
99.

1. People here are one injury away from starvation, one misspoken word away from detainment or death.
2. Soon, however, she notices the lack of access to basic medical care or education.
3. Life in a rural Kashmiri village seems idyllic to Shalini at first, as she's befriending lovely people and admiring majestic natural scenery, especially in contrast to the cacophony of urban Mumbai.
4. Moreover, the ever-present political disruptions mean that life in Kashmir is far from a Shangri-La utopia.

Solution: A good way to look for the introductory sentence is to eliminate the incorrect choices. Sentence 4 starting with "Moreover', sentence 2 starting with "Soon" and sentence 1 talking about "People here.." can't be introductory statements to any given paragraph. On the other hand, sentence 3, "Life in rural...Mumbai" beautifully introduces Shalini and her first impression of the village. Sentence 3 and 2 are mandatory pairs as these talk about her. Again sentence 1 and 4 will be paired sentences as these refer to the life in Kashmir. Hence 3214
100.

1. The study, published in the Lancet recently, revealed that people living in democratic countries live longer than those who don't; they also have less of a chance of dying from heart disease, strokes, and even road accidents.
2. Incredible as it may sound, we are now told that democracy is not just good for the soul, it is good for the body too.
3. Without pressure from voters or foreign-aid agencies, dictators have less incentive to finance more expensive prevention and treatment of heart disease, cancers, and other chronic illnesses.
4. The study suggests that elections and the health of the people are increasingly inseparable.
5. A study spanning 170 countries found a strong correlation between health and the most form of government.

Solution: Sentence 2, "We are now told.." sets the tone of the passage. Sentence 5, "A study spanning.. government." takes the idea forward. Sentence 5 and 1 are paired sentences as the former exemplifies the latter. This will be followed by statement 4 as it further talks about the study and what it entails. Hence, 25143

