E-BOOK
ELEMENTARY ALGEBRA

SuperGrads Study Material

Part of the most Comprehensive Classroom Training, Prep Content & Test Series across the Nation.

QUANTITATIVE ABILITY

ELEMENTARY ALGEBRA

Properties of surds:

$\left[\sqrt[n]{a}\right]^n = a.$

$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}.$

$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}.$

Laws of Indices:

If a and b are non-zero rational numbers and m and n are rational numbers then,

$a^0 = 1.$

$a^{-m} = \dfrac{1}{a^m}.$

$\sqrt[m]{a} = a^{\left(\frac{1}{m}\right)}.$

$a^{m/n} = \sqrt[n]{a^m}.$

$a^m \times a^n = a^{m+n}.$

$a^m \div a^n = a^{m-n}.$

$(a^m)^n = a^{mn}.$

$(ab)^m = a^m b^m.$

$a^{m^n} = a^{(m^n)} = a$ raised to the power (*m raised to the power n*).

If $a^m = a^n$, then m = n.

If $a^m = b^m$ and m ≠ 0, then a = b if m is odd and a = $\pm$b if m is even.

Laws of Logarithms:

✓ $\log_b 1 = 0$

✓ $\log_a a = 1.$

✓ $\log_a b \times \log_b a = 1$

✓ $\log_b(m \times n) = \log_b m + \log_b n.$

✓ $\log_b\left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

✓ $\log_b m^n = n\log_b m.$

✓ $\log_b m = \dfrac{\log_a m}{\log_a b} = \log_a m \times \log_b a$

✓ $b^{\log_b n} = n.$

✓ If $\log_a m = \log_b n$ and if m = n, then a will be equal to b.

✓ If $\log_a m = \log_b n$ and if a = b, then m will be equal to n.

Binomial Theorem:

If n is a natural number that is greater than or equal to 2, then according to the binomial theorem:

$(x + a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + {}^nC_3\, x^{n-3} a^3 + ... + {}^nC_n\, x^0 a^n.$

Here, ${}^nC_r = \dfrac{n!}{(n-r)!\, r!}.$

Roots of Quadratic Equation:

The two roots of two quadratic equation, $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Algebraic Formula:

$(a + b)(a - b) = a^2 - b^2$.

$(a + b)^2 = a^2 + 2ab + b^2$.

$(a - b)^2 = a^2 - 2ab + b^2$.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

PROGRESSION

Arithmetic Progression:

$T_n = a + (n - 1)d$.

$S_n = \frac{n}{2}[2a + (n - 1)d]$.

Geometric Progression:

$T_n = ar^{n-1}$. $S_n = \frac{a(r^n - 1)}{(r - 1)}$.

$S_\infty = \frac{a}{1 - r}$ for $r < 1$.

Harmonic Progression:

$T_n = \frac{1}{a + (n - 1)d}$

SUM OF IMPORTANT SERIES

Sum of first n natural numbers:

$1 + 2 + 3 + 4 + ... + n = \frac{n(n + 1)}{2}$.

Sum of the squares of the first n natural numbers:

$1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n + 1)(2n + 1)}{6}$.

Sum of the cubes of the first n natural numbers:

$1^3 + 2^3 + 3^3 + 4^3 + ... + n^3 = \left[\frac{n(n + 1)}{2}\right]^2$.

MODERN MATH

Factorial:

$n! = 1 \times 2 \times 3 \times ... \times (n - 1)n$.

$n! = n \times (n - 1)!$

Permutations:

$^nP_r = \frac{n!}{(n - r)!}$.

Combinations:

$^nC_r = \frac{n!}{(n - r)!r!}$.

Important Properties:

$^nC_r = {}^nC_{n-r}$

$^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n$

Partition Rule:

Number of ways of distributing n identical things among r person when each person may get any number of things = $^{n+r-1}C_{r-1}$.

Probability:

$$\text{Probability of an event } = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}.$$

$$\text{Odds in favor } = \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}.$$

$$\text{Odds against } = \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}.$$