

OPERATIONS ON NUMBER

➤ Divisibility Rules:

A number is divisible by 2 if it is an even number.

A number is divisible by 3 if the sum of the digits is divisible by 3.

A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

A number is divisible by 5 if the units digit is either 5 or 0.

A number is divisible by 6 if the number is also divisible by both 2 and 3.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

A number is divisible by 9 if the sum of the digits is divisible by 9.

A number is divisible by 10 if the units digit is 0.

A number is divisible by 11 if the difference of the sum of its digits at odd places and the sum of its digits at even places, is divisible by 11.

A number is divisible by 12 if the number is also divisible by both 3 and 4.

POINT TO REMEMBER FOR SPECIAL CASE -

For Divisibility of 7 - We take Unit digit & multiply with **2** then **Subtract** .

For Divisibility of 13 - We take Unit digit & multiply with **4** then **Add** .

For Divisibility of 17 - We take Unit digit & multiply with **5** then **Subtract** .

For Divisibility of 19 - We take Unit digit & multiply with **2** then **Add** .

A number is divisible by 7 if it Follows the below rules:

First of all we recall the osculator for 7. Once again , for your convenience , as $7 \times 3 = 21$ (One More than 2 X 10), our negative osculator is 2 . This Osculator '2' is our key - digit . This and only this digit is used to check the divisibility of any number by 7.

See how it works -

Ex. Is 112 divisible by 7 ?

Step I : $112 : 11 - 2 \times 2 = 7$ (Separate the last digit & multiply with two & then subtract)

Here we can see 7 Is divisible by 7, then we can say 112 is also divisible by 7.

Ex. Is 2961 divisible by 7?

Step I : $296 - 1 \times 2 = 294$

Step II : $29 - 4 \times 2 = 21$.

Here we can see 21 Is divisible by 7, then we can say 2961 is also divisible by 7 .

Note : Same Process will be applicable for Bigger Number .

A number is divisible by 13 if it Follows the below rules:

Ex. Is 143 divisible by 13 ?

Step I : $14 + 3 \times 4 = 26$.

Here we can see 26 Is divisible by 13, then we can say 143 is also divisible by 13 .

A number is divisible by 17 if it Follows the below rules

Ex. Is 1904 divisible by 17 ?

Step I : $190 - 4 \times 5 = 170$.

Here we can see 170 Is divisible by 17 . then we can say 1904 is also divisible by 17 .

A number is divisible by 19 if it Follows the below rules

Ex. Is 149264 divisible by 19 ?

Step I : $14926 + 4 \times 2 = 14934$.

Step II : $1493 + 4 \times 2 = 1501$

Step III : $150 + 1 \times 2 = 152$

Step IV : $15 + 2 \times 2 = 19$

Here we can see 19 Is divisible by 19 . then we can say 149264 is also divisible by 19 .

HCF & LCM OF NUMBERS

H.C.F: It Stands for Highest Common Factor / Greatest Common Divisor (G.C.D) and Greatest Common Measure (G.C.M).

L.C.M : It Stands for Lowest Common Factor / Lowest Common Divisor (L.C.D) and Lowest Common Measure (L.C.M).

- The H.C.F. of two or more numbers is the greatest number that divides each one of them exactly.
- The least number which is exactly divisible by each one of the given numbers is called their L.C.M.
- Two numbers are said to be co-prime if their H.C.F. is 1.

H.C.F. of fractions = H.C.F. of numerators/L.C.M of denominators

L.C.M. of fractions = G.C.D. of numerators/H.C.F of denominators

Product of two numbers = Product of their H.C.F. and L.C.M.

SIMPLIFICATION

BODMAS Rule: This Rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression .

Here, **B – Bracket**

O – Of

D – Division

M – Multiplications

A – Addition

S – Subtractions

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

➤ $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

➤ $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

ALGEBRA

Sum of first n natural numbers = $n(n+1)/2$

Sum of the squares of first n natural numbers = $n(n+1)(2n+1)/6$

Sum of the cubes of first n natural numbers = $[n(n+1)/2]^2$

Sum of first n natural odd numbers = n^2

Average = (Sum of items)/Number of items

➤ Arithmetic Progression -

An A.P. is of the form $a, a+d, a+2d, a+3d, \dots$

where a is called the 'first term' and d is called the 'common difference'

n th term of an A.P. $t_n = a + (n-1)d$

Sum of the first n terms of an A.P. $S_n = n/2[2a+(n-1)d]$ or $S_n = n/2(\text{first term} + \text{last term})$

➤ Geometrical Progression (G.P) -

A G.P. is of the form a, ar, ar^2, ar^3, \dots

where a is called the 'first term' and r is called the 'common ratio'.

n th term of a G.P. $t_n = ar^{n-1}$

Sum of the first n terms in a G.P. $S_n = a|1-r^n|/|1-r|$

PERMUTATION AND COMBINATION

Factorial Notation

Let n be a positive integer. Then, factorial n , denoted $n!$ is defined as:

$$n! = n(n - 1)(n - 2) \dots 3.2.1.$$

➤ POINTS TO REMEMBER

$0! = 1.$

$1! = 1.$

$2! = 2.$

- 3! = 6.
- 4! = 24.
- 5! = 120.
- 6! = 720.
- 7! = 5040.
- 8! = 40320.
- 9! = 362880.

➤ **Permutations:**

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

➤ **Examples:**

All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb).

All permutations made with the letters a, b, c taking all at a time are: (abc, acb, bac, bca, cab, cba)

➤ **Number of Permutations**

Number of all permutations of n things, taken r at a time, is given by:

$${}^n P_r = n(n - 1)(n - 2) \dots (n - r + 1) = \frac{n!}{(n - r)!}$$

➤ **Examples:**

$${}^6 P_2 = (6 \times 5) = 30.$$

$${}^7 P_3 = (7 \times 6 \times 5) = 210.$$

Cor. number of all permutations of n things, taken all at a time = n!.

➤ **An Important Result:**

If there are n subjects of which p₁ are alike of one kind; p₂ are alike of another kind; p₃ are alike of third kind and so on and p_r are alike of rth kind, such that (p₁ + p₂ + ... p_r) = n.

Then, number of permutations of these n objects is = $\frac{n!}{(p_1!)(p_2!) \dots (p_r!)}$

Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a combination.

Examples:

Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA. Note: AB and BA represent the same selection.

All the combinations formed by a, b, c taking ab, bc, ca.

The only combination that can be formed of three letters a, b, c taken all at a time is abc.

Various groups of 2 out of four persons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Note that ab ba are two different permutations but they represent the same combination.

➤ **Number of Combinations:**

The number of all combinations of n things, taken r at a time is:

$${}^n C_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2) \dots \text{to } r \text{ factors}}{r!}$$

Note:

${}^n C_n = 1$ and ${}^n C_0 = 1$.

${}^n C_r = {}^n C_{(n-r)}$

Examples:

1. ${}^{11} C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$.

2. ${}^{16} C_{13} = {}^{16} C_{(16-13)} = {}^{16} C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$.

PROBABILITY

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

Rolling an unbiased dice.

Tossing a fair coin.

Drawing a card from a pack of well-shuffled cards.

Picking up a ball of certain colour from a bag containing balls of different colours.

Details:

- i. When we throw a coin, then either a Head (H) or a Tail (T) appears.
- ii. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.
- iii. A pack of cards has 52 cards.

It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.

Cards of spades and clubs are **black cards**.

Cards of hearts and diamonds are **red cards**.

There are 4 honours of each unit.

There are **Kings, Queens and Jacks**. These are all called **face cards**.

➤ **Sample Space:**

When we perform an experiment, then the set S of all possible outcomes is called the **sample space**.

➤ **Examples:**

In tossing a coin, $S = \{H, T\}$

If two coins are tossed, the $S = \{HH, HT, TH, TT\}$.

In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.

Event:

Any subset of a sample space is called an **event**.

Probability of Occurrence of an Event:

Let S be the sample and let E be an event.

Then, $E \subseteq S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

Results on Probability:

$$P(S) = 1$$

$$0 \leq P(E) \leq 1$$

$$P(\Phi) = 0$$

For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

AVERAGE

Average : (Sum of Observation / Number of Observations)

Suppose a Man cover a certain Distance at X kmph and an equal distance at Y kmph . Then , the average speed during the whole journey is $[\frac{2XY}{X+Y}]$

SURDS AND INDICES

LAWS OF INDICES :

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^0 = 1$$

SURDS : Let a be rational number and n be a positive integer such that $a^{(1/n)} = a$
Then, a is called a surd of order n .

LAWS OF SURDS :

$$a = a^{(1/n)}$$

$$ab = a \times b$$

$$\sqrt[n]{\frac{a}{b}} = \frac{a}{b}$$

$$(a)^n = a$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$(a)^m = a^m$$

PERCENTAGE

To express $x\%$ as a fraction: We have, $x\% = \frac{x}{100}$

To express $\frac{a}{b}$ as a percentage : We have, $\frac{a}{b} \% = \left(\frac{a}{b} \times 100\right)$

If A is $R\%$ more than B, then B is less than A by $R / (100+R) \times 100$

If A is $R\%$ less than B, then B is more than A by $R / (100-R) \times 100$

Population after n years : $P(1 + R / 100)^n$

Population before n years : $P(1 - R/100)^n$

If the price of a commodity increases by R%, then reduction in consumption, not to increase the expenditure is : $R/(100+R) \times 100$

If the price of a commodity decreases by R%, then the increase in consumption, not to decrease the expenditure is : $R/(100-R) \times 100$

Value of Machine after n years : $P(1 - R/100)^n$

Value of Machine before n years : $P(1 + R/100)^n$

PROFIT AND LOSS

Gain = Selling Price(S.P.) - Cost Price(C.P)

Loss = Cost Price (C.P.) - Selling Price (S.P)

Gain % = $\text{Gain} \times 100 / \text{C.P}$

Loss % = $\text{Loss} \times 100 / \text{C.P}$

S.P. = $[(100 + \text{Gain}\%)/100] \times \text{C.P}$

S.P. = $[(100 - \text{Loss}\%)/100] \times \text{C.P}$

C.P. = $[100/ (100 + \text{Gain}\%)] \times \text{S.P}$

C.P. = $[100/ (100 - \text{Loss}\%)] \times \text{S.P}$

When a person sell two similar items , one at a gain of say x% , and other at a loss of x% then the seller always incur a loss given by - $\text{Loss \%} = (\text{Common loss \& gain \%} / 10)^2$

If a trader professes to sell his goods at cost price , but uses false weight , then $\text{Gain}\% = [\text{Error} / (\text{True value} - \text{Error})] \times 100 \%$

TRUE DISCOUNT

Ex. Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to R. 156 in 4 years. So, the payment of Rs. now will clear off the debt of Rs. 156 due 4 years hence. We say that:

Sum due = Rs. 156 due 4 years hence:

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 - 100) = Rs. 56 = (Sum due) - (P.W.)

We define: T.D. = Interest on P.W.;

Amount = (P.W.) + (T.D.)

Interest is reckoned on P.W. and true discount is reckoned on the amount.

Let rate = R% per annum and Time = T years. Then,

$$P.W. = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times T.D.}{R \times T}$$

$$T.D. = \frac{(P.W.) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$\text{Sum} = \frac{(S.I.) \times (T.D.)}{(S.I.) - (T.D.)}$$

(S.I.) - (T.D.) = S.I. on T.D.

When the sum is put at compound interest, then $P.W. = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$

RATIO & PROPORTIONS

The ratio a : b represents a fraction a/b. a is called antecedent and b is called consequent.

The equality of two different ratios is called proportion.

If a : b = c : d then a, b, c, d are in proportion. This is represented by a : b :: c : d.

In a : b = c : d, then we have a * d = b * c.

If a/b = c/d then (a + b) / (a - b) = (d + c) / (d - c).

TIME & WORK

If A can do a piece of work in n days, then A's 1 day's work = 1/n

If A and B work together for n days, then (A+B)'s 1 days's work = 1/n

If A is twice as good workman as B, then ratio of work done by A and B = 2:1

➤ **Basic Formula :**

If M1 men can do W1 work in D1 days working H1 hours per day and M2 men can do W2 work in D2 days working H2 hours per day (where all men work at the same rate), then

$$M1 D1 H1 / W1 = M2 D2 H2 / W2$$

If A can do a piece of work in p days and B can do the same in q days, A and B together can finish it in pq / (p+q) days

PIPES & CISTERNS

If a pipe can fill a tank in x hours, then part of tank filled in one hour = $1/x$

If a pipe can empty a full tank in y hours, then part emptied in one hour = $1/y$

If a pipe can fill a tank in x hours, and another pipe can empty the full tank in y hours, then on opening both the pipes,

the net part filled in 1 hour = $(1/x - 1/y)$ if $y > x$

the net part emptied in 1 hour = $(1/y - 1/x)$ if $x > y$

TIME & DISTANCE

Distance = Speed X Time

1 km/hr = $5/18$ m/sec

1 m/sec = $18/5$ km/hr

Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $2xy/(x+y)$ kmph.

PROBLEMS ON TRAINS

- ✓ Time taken by a train x metres long in passing a signal post or a pole or a standing man is equal to the time taken by the train to cover x metres.
- ✓ Time taken by a train x metres long in passing a stationary object of length y metres is equal to the time taken by the train to cover $x+y$ metres.
- ✓ Suppose two trains are moving in the same direction at u kmph and v kmph such that $u > v$, then their relative speed = $u-v$ kmph.
- ✓ If two trains of length x km and y km are moving in the same direction at u kmph and v kmph, where $u > v$, then time taken by the faster train to cross the slower train = $(x+y)/(u-v)$ hours.
- ✓ Suppose two trains are moving in opposite directions at u kmph and v kmph. Then, their relative speed = $(u+v)$ kmph.
- ✓ If two trains of length x km and y km are moving in the opposite directions at u kmph and v kmph, then time taken by the trains to cross each other = $(x+y)/(u+v)$ hours.
- ✓ If two trains start at the same time from two points A and B towards each other and after crossing they take a and b hours in reaching B and A respectively, then A's speed : B's speed = $(\sqrt{b} : \sqrt{a})$

- ✓ Speed of Train = (Sum of the length of two trains) / Time taken
- ✓ Time taken to cross a stationary Engine = (Length of the train + Length of engine) / Speed of the train .
- ✓ Time taken to Cross a signal Post = Length of the train / Speed of the Train

BOATS AND STREAM

In water, the direction along the stream is called downstream. And, the direction against the stream is called upstream.

If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then :

Speed downstream = $(u + v)$ km/hr

Speed upstream = $(u - v)$ km/hr

If the speed downstream is a km/hr and the speed upstream is b km/hr, then :

Speed in strill water = $1/2 (a + b)$ km/hr

Rate of stream = $1/2 (a - b)$ km/hr

SIMPLE AND COMPOUND INTEREST

Let P be the principal, R be the interest rate % Per annum, and N be the time period.

Simple Interest = $(P \times N \times R)/100$

Compound Interest = $P(1 + R/100)^N - P$

Amount = Principal + Interest

Let Principal = P , Rate = $R\%$ per annum, Time = n years.

When interest is compound Annually:

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

When interest is compounded Half-yearly:

$$\text{Amount} = P \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

When interest is compounded Quarterly:

$$\text{Amount} = P \left[1 + \frac{(R/4)}{100} \right]^{4n}$$

When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

When Rates are different for different years, say R1%, R2%, R3% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right).$$

Present worth of Rs x due n years hence is given by:

$$\frac{x}{\left(1 + \frac{R}{100} \right)^n}.$$

AREA AND VOLUME

Some Basic Concept –

- Sum of the angle of a triangle is = 180 degree
- The sum of any two side of a triangle is greater than the third side .
- Pythagorous Theorem = Hypotenuse² = (Base)² + (Height)²
- The line Joining the mid point of a side of a triangle to the opposite vertex is called the median .
- In an Isoscles triangle , the altitude from the vertex bisects the base
- The median of a triangle divide it into two triangle of the same area .
- The area of the triangle formed by joining the mid points of the side of a given triangle is one forth of the area of the given triangle .
- The diagonals of a parallelogram bisect each other.
- Each diagonal of a parallelogram divides it into triangles of the same area.
- The diagonals of a rectangle are equal and bisect each other.
- The diagonals of a square are equal and bisect each other at right angles.
- The diagonals of a rhombus are unequal and bisect each other at right angles.
- A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
- Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

Area of a rectangle = (Length x Breadth).

$$\therefore \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}} \right) \quad \text{and Breadth} = \left(\frac{\text{Area}}{\text{Length}} \right).$$

Perimeter of a rectangle = 2 (Length + Breadth)

Area of a square = (side)² = 1/2 (diagonal)².

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ (Side)²

Area of 4 walls of a room = 2 (Length + Breadth) x Height.

Area of a triangle = $\frac{1}{2}$ x Base x Height.

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
 where a, b, c are the sides of the triangle and $s = 1/2(a + b + c)$.

Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

Radius of Circum-circle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

Radius of incircle of a triangle of area Δ and semi-perimeter $r = \frac{\Delta}{s}$.

Area of parallelogram = (Base x Height).

Area of a rhombus = $\frac{1}{2}$ x (Product of diagonals).

Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x distance between them.

Area of a circle = πR^2 , where R is the radius.

Circumference of a circle = $2\pi R$.

Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle.

Area of a sector = $\frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2 \theta}{360}$.

Circumference of a semi-circle = πR .

CUBOID

Let length = l , breadth = b and height = h units. Then

Volume = $(l \times b \times h)$ cubic units.

Surface area = $2(lb + bh + lh)$ sq. units.

Diagonal = $\sqrt{(l^2 + b^2 + h^2)}$ units.

CUBE

Let each edge of a cube be of length a . Then,

Volume = a^3 cubic units.

Surface area = $6a^2$ sq. units.

Diagonal = $\sqrt{3}a$ units.

CYLINDER

Let radius of base = r and Height (or length) = h . Then,

$$\text{Volume} = (\pi r^2 h) \text{ cubic units.}$$

$$\text{Curved surface area} = (2\pi rh) \text{ sq. units.}$$

$$\text{Total surface area} = 2\pi r(h + r) \text{ sq. units.}$$

CONE

Let radius of base = r and Height = h . Then,

$$\text{Slant height, } l = \sqrt{(h^2 + r^2)} \text{ units.}$$

$$\text{Volume} = \left(\frac{1}{3}\pi r^2 h\right) \text{ cubic units.}$$

$$\text{Curved surface area} = (\pi rl) \text{ sq. units.}$$

$$\text{Total surface area} = (\pi rl + \pi r^2) \text{ sq. units.}$$

SPHERE

Let the radius of the sphere be r . Then,

$$\text{Volume} = \left(\frac{4}{3}\pi r^3\right) \text{ cubic units.}$$

$$\text{Surface area} = (4\pi r^2) \text{ sq. units.}$$

HEMISPHERE

Let the radius of a hemisphere be r . Then,

$$\text{Volume} = \left(\frac{2}{3}\pi r^3\right) \text{ cubic units.}$$

$$\text{Curved surface area} = (2\pi r^2) \text{ sq. units.}$$

$$\text{Total surface area} = (3\pi r^2) \text{ sq. units.}$$

Note: 1 litre = 1000 cm³.

GEOMETRY – SECTION**TRIGNOMETRY****TRIGNOMETRIC FORMULA:**

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \operatorname{cosec}^2(x)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = 2 \cos^2(x) - 1$$

$$\sin(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(2x) = 1 - 2 \sin^2(x)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \tan(y)}$$

$$\sin(x) \times \sin(y) = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$

$$\cos(x) \times \cos(y) = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$$

$$\sin(x) \times \cos(y) = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos(x) \times \sin(y) = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

$$\sin\theta \times \operatorname{cosec}\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

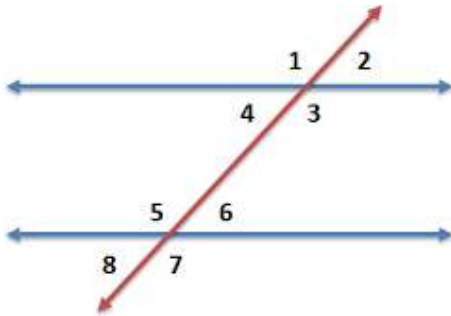
$$\cos^2\theta = 1 - \sin^2\theta$$

TRIGONOMETRIC VALUES:

Degrees	0°	30°	45°	60°	90°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\tan \theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	NA	0

BASIC GEOMETRY FORMULA

PARALLEL LINES



Corresponding angles are equal i.e : $1=5$, $2=6$, $4=8$, $3=7$

Alternate interior angles are equal i.e : $4=6$, $5=3$

Alterate Exterior angle are equal : $2=8$, $1=7$

In other words interior angles same side = 2 right angles = $180^\circ = \pi$ radians = $1/2$ turn

Sum of exterior angles same side $\angle 2 + \angle 7 = 180^\circ$

Types of Angle

Acute angle = $0^\circ - 90^\circ$

Right Angle = 90°

Obtuse angle = $90^\circ - 180^\circ$

Straight Angle = 180°

Reflex Angle = $180^\circ - 360^\circ$

Complete angle = 360°

Complementary Angle = sum of two angles = 90°

Supplementary angle = sum of two angles = 180°

Triangle

Vertices A, B, C

Angles = $\angle A$, $\angle B$, $\angle C$

Three sides AB, BC, AC

Triangle two Types

A. Based on sides

Equilateral Triangle : All three sides equal

Isosceles Triangle : Two sides equal

Scalene Triangle : all three sides different

B. Based on Angles

Right Angle Triangle : One angle 90°

Obtuse Angle Triangle : One angle more than 90°

Acute Angle Triangle : All angles less than 90°

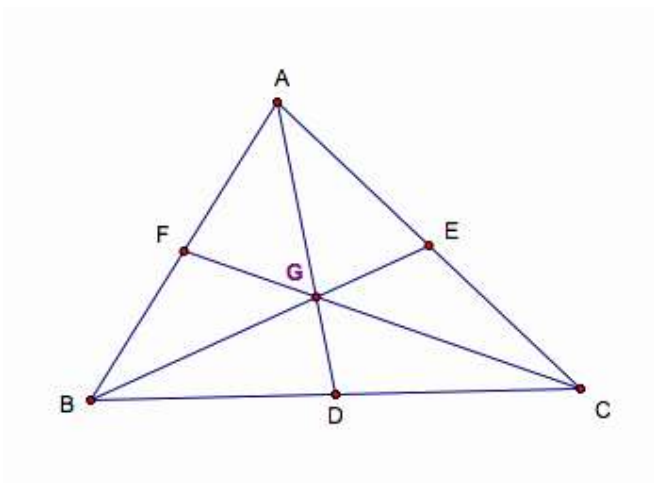
When $AC^2 < AB^2 + BC^2$ (Acute angle triangle)

When $AC^2 > AB^2 + BC^2$ (Obtuse angle triangle)

When $AC^2 = AB^2 + BC^2$ (Right angle triangle)

CENTER OF TRIANGLE

A. CENTROID



A median divides triangle into 2 equal parts

$$AG : GD = 2:1$$

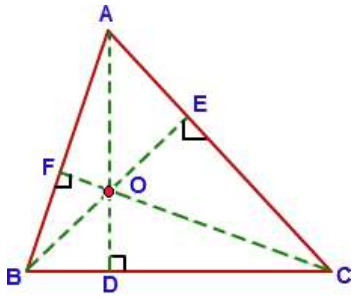
$$BG : GB = 2:1$$

$$CG : GF = 2:1$$

$2 \times (\text{Median})^2 + 2 \left(\frac{1}{2} \text{ the third side}\right)^2 = \text{Sum of the square of other sides .}$

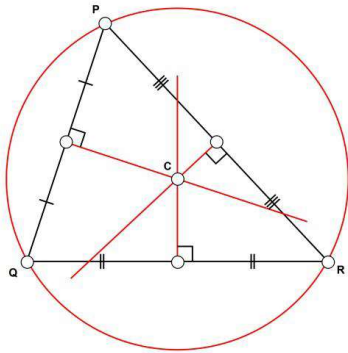
$$2AD^2 + 2 \times (BC/2)^2 = (AB)^2 + (AC)^2.$$

B. ORTHOCENTER



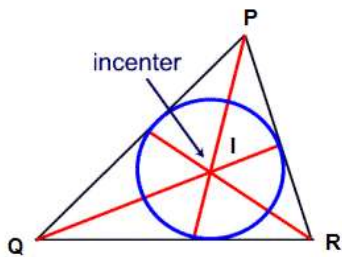
$$\angle A + \angle BOC = 180 \text{ Degree} = \angle C + \angle AOB = \angle B + \angle AOC$$

C. CIRCUMCENTER



$$\angle QCR = 2\angle QPR$$

D. INCENTER

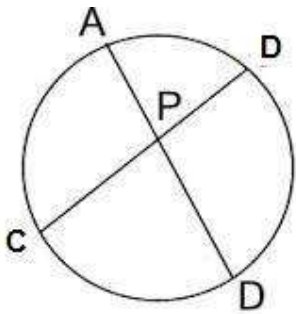


$$\angle QIR = 90 + \frac{1}{2} \angle P$$

$\angle QIR = 90 - \frac{1}{2} \angle P$ if QI + RI be the angle bisector of exterior angles at Q & r .

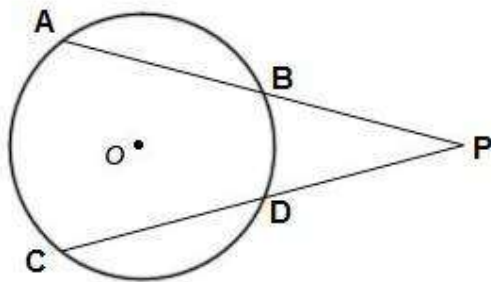
CHORD OF A CIRCLE

Case – I When Intersect Internally



$$PA \times PB = PC \times PD$$

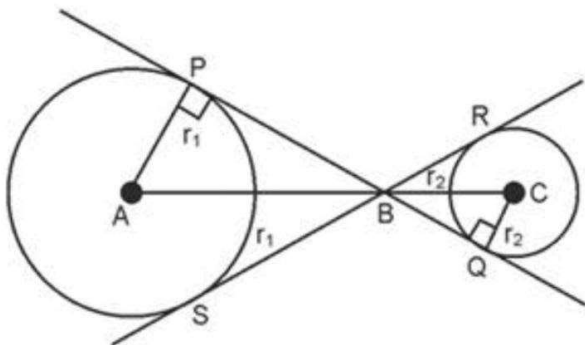
Case – II When Intersect Externally



$$PA \times PB = PC \times PD$$

TANGENTS

Case – I In-Direct Common Tangent / Transverse Common Tangent

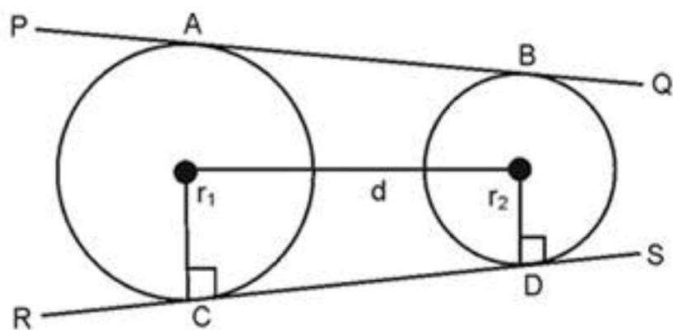


$$AB : BC = r_1 : r_2$$

Assume $AC =$ Distance between centres $= d$

$$PQ^2 = RS^2 = d^2 - (r_1 + r_2)^2$$

Case – II Direct Common Tangent



$$CD^2 = AB^2 = d^2 - (r_1 - r_2)^2$$